# Amount of Prestressing Based on Serviceability Requirements

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## Summary
The paper proposes design criteria for the choice of the amount of prestressing. Based on an extension of the well known load balancing method, the proposed criteria permit a simple and effective design of the amount of prestressing at serviceability. The paper also discusses the favourable effect of compressive stresses induced by prestressing on cracking and water-tightness of PC structures. In situ measurements as well as extensive test results demonstrate that the proposed criteria lead to structures that are more durable and less prone to an increase in deflections and cracking over time.

**Keywords:** bridges, prestressed concrete, serviceability, cracking, design criteria, cyclic loading, in situ measurements, residual crack opening, water-tightness

## 1. Introduction
Behaviour in service is extremely important for all structures, and serviceability requirements should be central in the choice of the amount of prestressing, particularly for bridges. They are unfortunately often omitted in the initial design, only to be checked in subsequent verifications. This is unfortunate, considering the fact that structures spend most of their useful life at the serviceability limit state. Owners and users alike are more affected by the actual behaviour of structures during their service life than by their possible behaviour at the ultimate limit state.

In spite of this, criteria based on ultimate limit state requirements are often used as the governing factor in the design of reinforced and prestressed concrete structures. There are advantages to that approach. A design based on ultimate state requirements leaves the designer a large freedom in the choice of the amount of prestressing to apply: if the design includes less prestressing, then a larger amount of passive reinforcement will be required. On the other hand, a design calling for more prestressing will lead to smaller amounts of passive reinforcement in order to satisfy ultimate limit state safety requirements. The final solution frequently combines these requirements with those of economics and ease of construction. In many cases, a smaller amount of prestressing may lead to a more economical solution, which is also easier to build.

One reason for this approach based on ultimate limit state requirements may be the lack of clear and easily applicable design procedures for the serviceability limit state. In spite of intense research in the field of cracking and deformations, there are few criteria for the choice of the amount a prestressing in relationship with serviceability requirements. This article proposes such criteria, based on the concept of compensation of deformations, which is essentially a generalszation of the load balancing method, which allows a simple choice of the required amount of prestressing, and on compressive stress values to ensure sufficiently small residual cracks under permanent loads..

Serviceability is normally controlled by behaviuor under permanent loads. Except for irreversible effects of cyclic loads that are considered below, the behaviour under variable service loads can usually be neglected, because :

- instantaneous cracking has limited consequences on durability and esthetics, provided the reinforcing steel does not yield,
- short-term deflection limits prescribed in codes are rarely controlling the design. As an example, Swiss regulations limit short-term deflections of bridges to 1/600 of the span. Typical concrete box girder bridges exhibit deflections of 1/1'500 to 1/3'000 of the span under maximum traffic loads.

As a consequence, reliable design criteria should not result from considerations under maximum service loads. Rather, the behaviour under permanent loads should be the focus of serviceability checks, because :

- permanently open through cracks that let water flow through the structure prevent it from remaining water-tight. Even when a waterproofing membrane is present on the deck, as is frequently the case in Europe, these cracks have detrimental consequences on the serviceability behaviour ;
- long-term deflections are primarily caused by permanent loads acting on the structure (loads and prestressing force) ;
- irreversible effects, induced by traffic loads and temperature gradients, while not important when they occur (see above), have a cumulative effect that increases the deflections in the permanent state. This is why such effects are usually added to the long-term deflections caused by permanent loads [1]. It has been observed that these contributions may induce a continuous increase in deflections, even after the 5 to 10 years it usually takes for creep to finish its development. In the case of large spans (> 100 m), this contribution can be in the amount of several centimeters [2]. These deflections are especially critical in cases where no camber can be applied, as is for example the case with incrementally launched bridges.

## 2. Concept of compensation of deformations

The concept of compensation of deformations is formulated as a generalisation of the load balancing method. Numerical values for the design criteria were derived from theoretical and experimental research, including extensive in situ measurements [1-7]. These values were further confirmed by studies of long-term cracking [8,12].

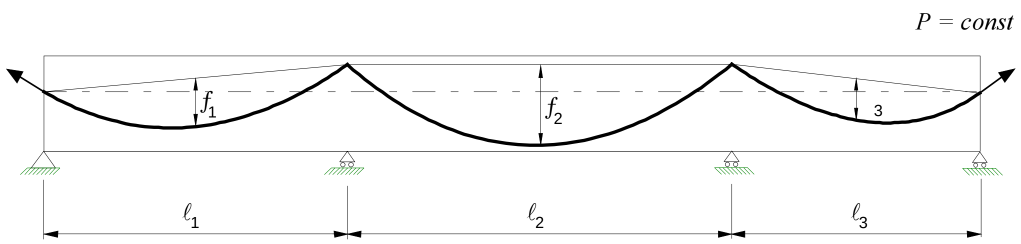


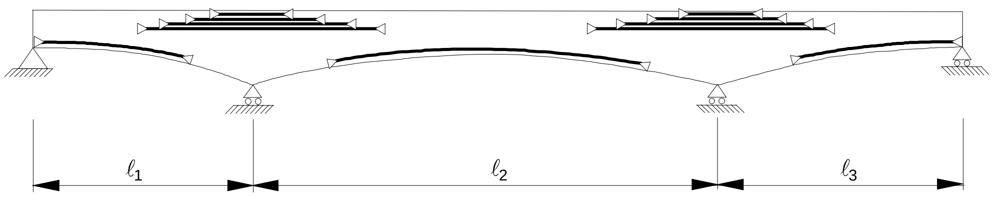

*Figure 1: Geometry and cable layout for a simple application of the load balancing method*

In the load balancing method applied to a beam, deviation forced induced by parabolic prestressing cables $u = 8\,fP/\ell^2$ (fig. 1) are directly opposing permanent loads $g$. The level of load balancing $\beta = u/g$ is thus defined as the ratio of the permanent load that is compensated by the deviation forces of the prestressing. Figure 1 shows a structure with a simple parabolic cable layout permitting an easy application of the load balancing method.

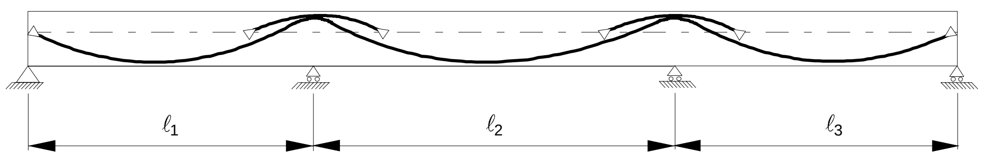
The application of the concept of load balancing is unfortunately limited in practice. If the geometry of the structure or the cable layout are complicated, as is frequently the case for bridges, it becomes impossible to characterise the entire structure by a single load balancing level β, because :

- the prestressing force is not constant along the structure
- the cable layout is not parabolic
- because the axis of gravity is not rectilinear.

Figure 2 shows cases in which the application of the load balancing method is not practical. In bridges built by free cantilevering (fig. 2a), the level of load balancing is zero, considering only the deck cables. These cables, however, because of their eccentricity with respect to the centroidal axis, and because this axis is not rectilinear, induce large moments and deflections of the structure, and effectively balance the dead load, making this construction method applicable. The load balancing method does not take into account the effect of the axial force, which is extremely important for structures that do not have a rectilinear axis of gravity. In practice, prestressed concrete bridges have a non-rectilinear axis of gravity, because section changes in the vicinity of the supports in general (increase of the web thickness or of the lower plate depth in box girder bridges).




a) bridge built by free cantilevering with contnuity cables




b) bridge with overlapping cables

*Figure 2: Situations for which the application of the load balancing method is impractical*

The whole concept of compensation of deformations stems from the desire to extend the simple approach of load balancing to a much wider range of structures. In the compensation of deformations approach, instead of *balancing loads*, the designer *compensates deformations*. Although it increases the computing effort, this approach has the great advantage of taking into account the effect of anchorage forces, the axial load induced by prestressing, and the effect of non-rectilinear gravity axes. The degree of compensation of deformations $\beta$ is thus defined as :

$$\beta = \frac{w_c\left(P_m\right)}{w_c\left(g\right)} \quad with \quad P_m = \frac{P_{t=0} + P_{t=\infty}}{2} \qquad (1)$$

where

$w_c$ : elastic deflection at mid-span (creep is not taken into account as it would appear in the numerator as well as in the denominator of eq. 1)
$P_m$ : average prestressing force over the lifetime of the structure
$\beta$ : degree of compensation of defections
$g$ : permanent and occasionally semi-permanent loads

Because the moments induced by the prestressing cables only approximate the moment diagram induced by permanent loads, the value of $\beta$ defined above varies slightly from point to point along the axis of the structure. For simplicity's sake, $\beta$ is normally taken at mid-span, as illustrated in fig. 3. It is the engineer's responsibility to adapt this simplifying assumption, which is perfectly acceptable for ordinary structures, if it is inappropriate for his problem.

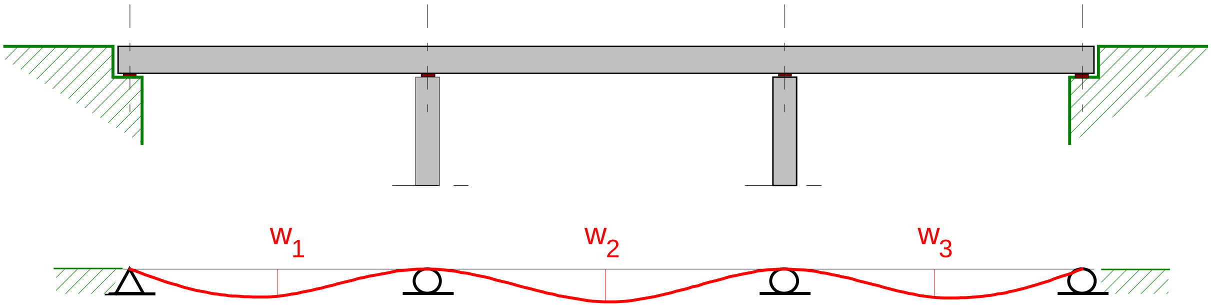



*Figure 3: Compensation of deflections for a three-spans continuous beam*

The method of compensation of deformations uses the same symbol $\beta$ that is commonly used for the level of load-balancing, because it constitutes a mere extension of this method. In the case of a simple structure such as shown in fig. 1, both approaches yield the same value for $\beta$.

## 3. Cracking

An analytical and experimental study is currently under way at the Swiss Federal Institute of Technology in Lausanne to further investigate the effect of cracking on structures subjected to cyclic actions [8,9]. This study investigates residual crack opening after a structure has been subjected to exceptional loads. Residual crack opening is defined as the opening of a crack, under

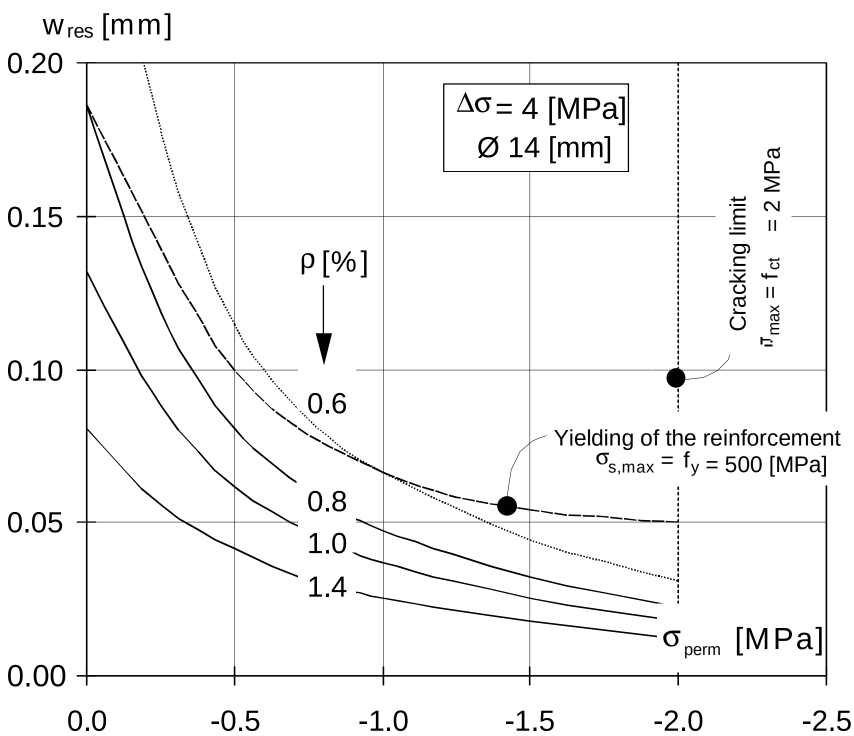



*Figure 4: Residual crack opening as a function of the permanent stress ($\sigma_{perm}$) and the reinforcement ratio ($\rho$) in the case of an exceptional tensile stress of $\sigma_{perm}$ + $\Delta\sigma$ with* $\Delta\sigma$ = 4 MPa.

permanent loads only. Typically, a properly prestressed structure is under compression under permanent loads, although cracks may occasionally open under exceptional loads. While it is necessary to consider such exceptional actions in the serviceability check, it would be unrealistic to design the whole structure to remain uncracked even under exceptional service loads. Rather, it is desirable to design the prestressing for a proper performance in the permanent state, and then consider the cumulative damaging effect of exceptional loads and its influence on the behaviour at the serviceability limit state.

Figure 4 shows typical results obtained from a large parametric study, obtained by a model [9] based on constitutive relationships developed in [10,11]. Passive reinforcement ratio, bar diameter and concrete properties have a strong influence on residual crack opening, but the two most important factors are the stress level under permanent loads and the magnitude of the maximal tensile stress under the exceptional load [13,14]. Table 1 summarises the results of this parametric study for typical stress ranges. It indicates the compressive stress required to effectively limit the residual crack opening under permanent loads $w_{res}$. Its validity is limited to the case when no yielding of the reinforcement occurs under the exceptional load.

*Table 1: Residual crack opening after an exceptional loading as a function of the permanent compressive stress. Independent from the reinforcement ratio and the maximal load, provided no yielding of the steel occurs.*

| $\sigma_{perm}$ [MPa] | ≈ 0 | -0.5 | -1 | -2 | ≤ -5 |
|---|---|---|---|---|---|
| $w_{res}$ [mm] | 0.2 | 0.1 | 0.075 | 0.05 | 0.025 |

Table 1 shows that the residual crack opening induced by a stress variation in a structure where the compressive stress under permanent loads is -2 MPa will be roughly 0,05 mm or 50 $\mu$m, hardly detectable.

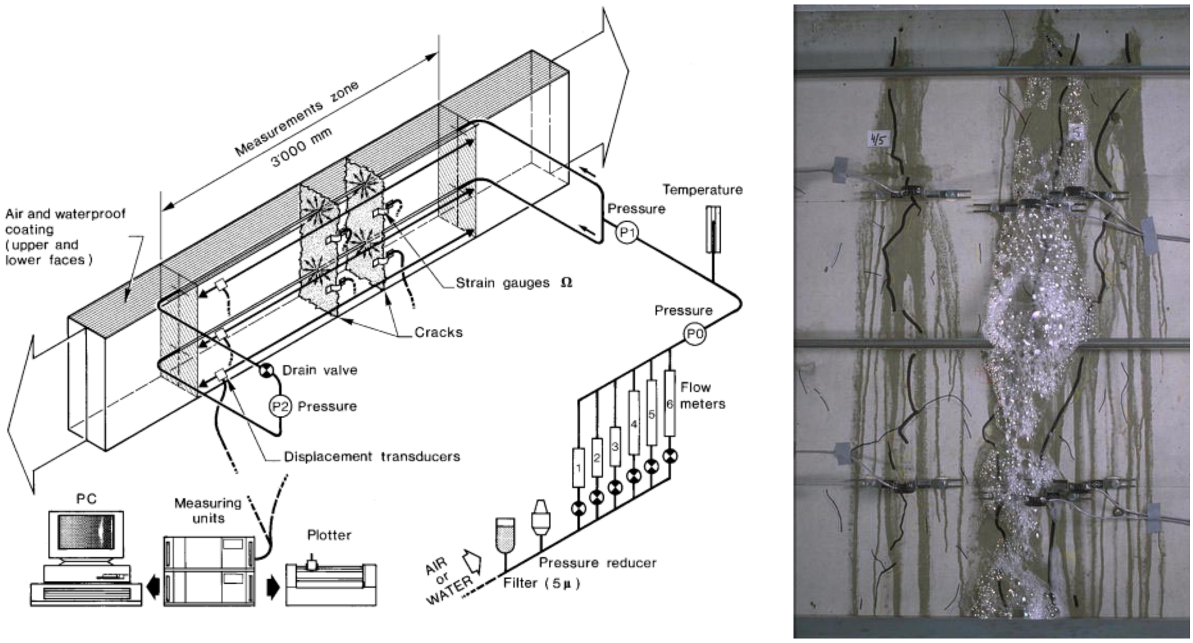



*Figure 5: General view of the test setup for flow measurement on large elements and detail of a crack with seeping water and foam*

A study was performed at the Swiss Federal Institute of Technology to quantify the effect of cracking on the water-tightness of reinforced concrete structures (fig. 5) [12,15,16,17,18]. A simple application of its conclusions can be made to the case of a residual crack opening resulting from an exceptional loading. Fig. 7a shows expressions for the calculation of the water flow $q_{smooth}$ through a simplified crack of width $w$, through a slab of thickness $h$, with perfectly smooth crack surfaces.

where :

$q_{smooth}$ : flow per m' of smooth plane [kg/s/m] ;
$\rho$ : volumic mass of the fluid [kg/m$^3$] ;
$\Delta p=$ : pressure difference over the length $h$ of the flow [Pa] ;
$\mu$ : dynamic viscosity of the fluid [Pa.s] ;
$h$ : thickness of the element, length of the through flow [m] ;
$w$ : distance between the two planes [m].

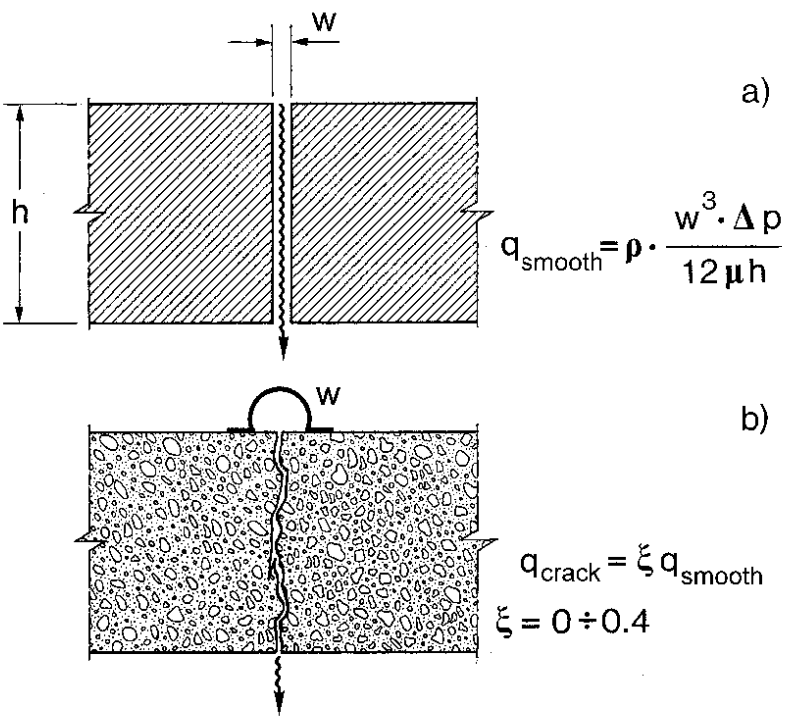



*Figure 7: Water flow between two planes*
*a) perfectly smooth planes (ideal case)*
*b) rough planes with a corrective coefficient ξ (case of a crack)*

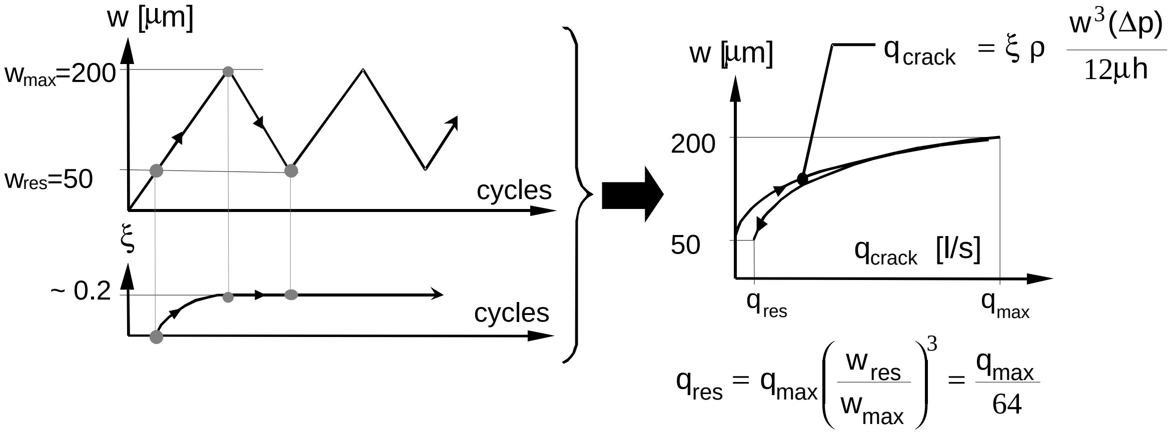



*Figure 6: Water flow through a crack for* $w_{max}$ = 200 μm *and* $w_{res}$ = 50 μm

Because of the rough texture of the surface of a crack, the water flow calculated for the ideal case $q_{smooth}$ needs to be reduced by a flow coefficient $\xi$. The flow $q_{crack}$ through an actual crack in concrete corresponds to values of $\xi$ between 0.0 and 0.4. This coefficient is a direct function of the crack opening at the surface of the concrete, and increases when crack opening increases (fig. 6). Experimental observation has shown that the flow coefficient does not decrease even if the crack partly closes under the effect of prestressing. It is apparently related to the irreversible damage induced by crack opening and is thus directly function of the maximal crack opening.

As a consequence, the maximum reduction of seepage flow for a residual crack is directly proportional to the cube of the ratio $w_{max}/w_{res}$. Of course, this approach is quite simplified, in that it does not consider self-sealing of crack due to hydration of free cement in the concrete, nor concrete expansion in the presence of water.

Although prestressing does not permit a complete re-closing of the cracks and cannot make a pre-cracked element fully water-tight, a residual crack opening of 50 $\mu m$ is practically undetectable and leads to almost no measurable water flow. An opening of 50 $\mu m$ was shown above to be a value typical of a prestressed element with a permanent compressive stress of 2 $N/mm^2$.

## 4. Recommendations for the choice of the amount of prestressing

The concept of compensation of deflections presented in section 2 and, for simple cases, the concept of load balancing, permit a choice of the amount of prestressing based on a single global and rational criterion : the level of compensation of deformations. As shown in [4] and [9], this concept takes into account crack control and limits permanent state deflections, while including some part of the effect of variable actions on the behaviour in service.

Table 2 gives recommended values of the degree of compensation of deformations as a function of the structural type and requirements. These values are indicative, in that they are to be used for the initial choice of the amount of prestressing, and that they are not intended to replace the usual code checks. As a rule, the amount of prestressing obtained by applying this table is slightly larger than the minimum amount required by most current codes.

| Type of structure | Increased requirements | Normal requirements |
|---|---|---|
| Road bridge | 0.9 | 0.8 |
| Rail bridge | 1.1 | 1.0 |
| Building slab | 0.6 | 0.5 |
| Heavily loaded slab | > 0.6 | > 0.5 |

*Table 2: Recommended β-values for various types of structures*

The degree of compensation of deformations required for bridges is close to unity, which has for consequence not only to limit the long term deflections, but also, because of the state of compressive stresses induced under permanent loads, to limit or completely prevent cracking. As seen in section 3, this leads to a very good water-tightness of the bridge deck, and therefore a good durability of the structure.

The values given for slabs are smaller than for beams, because, as a consequence of their shallow depth, slabs are noticeably influenced at the serviceability limit state by the tensile strength of concrete. Observed behaviour both short- and long-term related in [4] demonstrate that these values lead to an excellent performance in service. At the same time, bridges that had $\beta$-values significantly lower than indicated in table 2 often exhibited an unsatisfactory behaviour at service state.

## 5. Conclusions

Starting from a perspective of good behaviour at the serviceability limit state, very general criteria for the choice of the amount of prestressing have been derived. These criteria are focusing on the behaviour under permanent loads for both cracking and deflections. The influence of variable actions (movable loads and temperature) is taken into account through their irreversible contribution to the behaviour under permanent loads.

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