Comparison of the Amount of Reinforcement for a Box-girder Bridge Prestressed with Internal or External Cables

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Summary

A comparative study was conducted to highlight key differences in the required amounts of prestressing and passive reinforcement for externally and internally prestressed box-girder bridges. The reinforcement is designed on the basis of combined serviceability and structural safety criteria for varying span length and girder slenderness. Under the assumptions of the study, the externally prestressed box-girder is shown to require up to 25\% more prestressing force for girder heights around 2 m. At girder heights above 3.5 m however, internal prestressing becomes the more favorable type of prestressing. The study also shows that the externally prestressed girder tends to require more longitudinal and shear passive reinforcement in key design sections.

1. Introduction

The external prestressing of box-girder bridges has received increasing attention in recent years, both for new bridges and for the retrofitting of existing bridges \cite{1, 2}. German highway administration authorities have even given external prestressing the status of the preferred solution for new concrete box-girders \cite{1}. It has several key advantages over internal prestressing, as well as some disadvantages. One of the disadvantages is that it tends to require more reinforcement than a comparable internally prestressed bridge. This paper presents the findings of a numerical investigation conducted to compare the required amount of prestressed and passive reinforcement for a representative box-girder highway bridge prestressed with internal and external cables.

2. Description of the comparative study

The study was conducted to highlight key differences in terms of reinforcement between the two types of prestressing and to show how these differences vary with the span and slenderness of the girders. The actual detailed design of a prestressed concrete box-girder bridge requires a very significant effort. In order to make this study possible, several simplifying assumptions were made in the configuration of the bridges and in the design of the reinforcement. The design approach resembles the approach of a preliminary design.

2.1 Description of the bridge girders used in the comparative study

The comparative study was conducted on two box-girder road bridges illustrated in Fig. 1. The first girder is internally prestressed, the second is externally prestressed. Both are symmetrical five span girders with constant girder height. The side spans are 85\% of the length of the middle spans. The girders share the following material properties. The concrete quality is approximately equivalent to a C30/37 (EC 2). The yield resistance $f_y$ for the passive and prestressed reinforcement is 460 Mpa, respectively 1590 Mpa.
The cross-section of the internally prestressed girder was chosen to be representative of recent Swiss highway box-girder bridges. The top slab is 13.60 m wide with 3.50 m long cantilevers and a thickness varying between 0.25 m and 0.40 m (Fig. 1). The bottom slab thickness is 0.40 m at the support and 0.20 m in the field. The web width is 0.50 m.

The cross-section of the externally prestressed girder is similar to the internally prestressed section. It only differs in the position of the prestressing cables and the reduced girder web thickness (0.30 m). The comparative study presented below is based on the idea that the two girders are comparable, i.e. that their differences are limited to parameters directly linked to the type of prestressing (internal or external) with which they are prestressed.

The prestressing cables geometry is parabolic for the internally prestressed girder and polygonal for the externally prestressed girder (Fig.1). The external cables are deviated at the intermediary supports and twice in the spans. The position of the field deviators was chosen at the span third points (0.33 λ). The cables position at the intermediary supports was defined with \( a_{p,\text{sup}} \) equal to 0.18 m for internal prestressing and 0.45 m for external prestressing. The same values were used for the cables low position in the central span \( (a_{p,3}) \). In the other spans, the mid-span cable position was selected in order to produce the same \( \beta \) value (see section 2.2) in all the spans of the girders. The values of \( a_{p,1} \) and \( a_{p,2} \) depended on the girder depth. The cable geometry was selected to realistically reproduce the larger cable eccentricity possible with internal cables for a given girder height.

**Fig. 1 – Configuration of the bridges of the comparative parametric study.**
2.2 Design of the reinforcement

The approach to the reinforcement (preliminary) design for both internally and externally prestressed girders is illustrated in Fig. 2. The goal was to conduct a simplified but realistic design of the prestressing cables and of passive longitudinal and shear reinforcement in key sections. When code guidance was required, the current Swiss SIA structural codes were used [3]. Permanent loads consisted of the weight of the girder augmented with a 20 kN/m' dead load (the weight of the external prestressing deviators and cables was neglected). The traffic load consisted of a uniformly distributed load of 41.5 kN/m' and a concentrated load of 540 kN. Reinforcement design calculations at ultimate were performed with the prestressing "on the resistance side" for both internally and externally prestressed girders.

The amount of prestressing is chosen on the basis of a serviceability criteria. The criteria is the balancing of permanent load deflection which is described in detail in [4, 5]. The degree of load balancing $\beta$ has been shown to constitute a simple and reliable serviceability criteria for the selection and evaluation of the amount of prestressing for box-girder bridges. $\beta$ is calculated with $P_m$, the value of the prestressing force accounting for initial prestressing losses and half the long term losses. Extensive theoretical and experimental investigations have shown that a $\beta$ value of 0.8 results in satisfactory serviceability state behavior for standard concrete box girder highway bridges [5]. It is well suited both for the preliminary design and evaluation of both internally and externally prestressed bridges. A target value $\beta = 0.8$ was therefore chosen as a key fixed parameter of the comparative study. The required value of $P_m$ was calculated to obtain a balancing degree $\beta$ of 0.8 in all spans of the girder. The required section of prestressing steel could then be derived from $P_m$ assuming an initial cable tension stress of 0.70 $f_{tk}$ and 15 % long-term prestressing losses.

The amount of longitudinal passive reinforcement was calculated at mid-span and over the support. First a minimal reinforcement ratio is calculated on the basis of cracking control criteria (serviceability). Using current Swiss structural concrete codes, this resulted in a passive reinforcement ratio of 0.51 % in the girder's top and bottom slabs. Next, structural safety at flexure was verified, and if necessary, additional passive longitudinal reinforcement was calculated.

The amount of web shear reinforcement in the support region is calculated on the basis of a classic truss model used by the current Swiss structural concrete code ([3] with $\alpha = \alpha_0$). As for flexure, structural safety of the girder in shear was verified only under combined factored permanent and traffic load.
2.3 Approach

The study was conducted by comparing the findings of two parametric studies. The first parametric study consisted of calculating the required amount of prestressed and passive reinforcement for varying girder span length and slenderness for the internally prestressed bridge girder. A similar parametric study was conducted for the externally prestressed girder to allow a comparison of required prestressing and passive reinforcement in key sections. The comparison is performed for girder spans $\lambda$ ranging from 30 m to 80 m, and for girder slenderness $\lambda/h$ equal to 16, 22 and 28. These span and slenderness values cover most applications of prestressed concrete box-girder bridges. It is recognized that some of the combinations of span length and slenderness are not realistic, their interest is strictly in highlighting trends in the comparison.

The parametric studies were conducted using a simple analytical tool developed to calculate the girder passive and prestressed reinforcement for the different parameter combinations. The algorithm only accounts for flexural deformations in the calculation of the deflections. The computer program was written using Visual Basic programming language and is used in a spreadsheet environment. The tool is well adapted for parametric studies and the visualization and graphical exploitation of the results.

3. Results of the comparative study

The results of the study of comparable internally and externally prestressed highway box-girders are illustrated in Figs. 3 through 6. The comparison covers the required girder prestressing force and the amount of passive longitudinal and shear reinforcement in key sections.

3.1 Amount of prestressing

The amount of prestressing force $P_m$ required to satisfy the balancing criteria is shown in Fig. 3 for both types of prestressing. In the case of internal prestressing and for a given slenderness ratio, the required amount of prestressing force increases regularly with the span length. The behavior is different for external prestressing. The curves are flatter, meaning that the amount of prestressing does not decrease as much for smaller spans. For the (unrealistic) situation where the girder slenderness is 28 and the spans are below 45 m, the required prestressing force actually decreases as the span length increases. This behavior reflects the fact that at low girder depth, the externally prestressed girder is highly penalized by the reduced sag of its prestressing cable.

![Fig. 3 – Required prestressing force $P_m$ in function of the span length $\lambda$ for slenderness values of 16, 22 and 28.](image-url)
In Fig. 4, the curves of Fig. 3 for the internally and externally prestressed girders are superposed. The superposition shows that for span lengths below 40 m, external prestressing requires significantly more prestressing, even for low slenderness ratios. As the span length increases however, the advantage of internal prestressing in terms of prestressing force and section decreases, and a point is reached where internal prestressing becomes the more favorable type of prestressing. This results from the fact that at larger girder depths, the penalizing effect of the reduced cable sag of the external prestressing is more than compensated by the lighter weight (thinner webs) of the externally prestressed box girder. Fig. 4 shows that under the assumptions of this study, girders 4 m deep or deeper are more advantageous in the external prestressing configuration. For a slenderness of 16 for example, the crossing point is for a span length around 55 m.

3.2 Longitudinal Passive Reinforcement

The need for additional reinforcement to satisfy structural safety requirement is illustrated in Fig. 5. It shows whether the prestressing and passive reinforcement chosen on the basis of serviceability criteria (deflection balancing and crack-control) provide sufficient flexural capacity to the span. The structural safety of the girder is evaluated by comparing the required design flexural capacity of the middle span with its resisting flexural capacity. For the sake of simplicity, this evaluation is carried out on the basis of the beam plasticity theory using the ratio $M_{rd,min}/M_d$ (for the girder middle span, $M_d$ is the sum of the maximum support and field factored design moment and $M_{rd}$ is the sum of the ultimate flexural resistance of the mid-span and support sections divided by a resistance factor equal to 1.2). If the ratio $M_{rd,min}/M_d$ is above 1.0, and if the implied level of moment redistribution is allowable, the serviceability passive and prestressing reinforcements are sufficient for structural safety in flexure. If the ratio is below 1.0, it means that even with moment redistribution, the flexural reinforcement obtained by serviceability criteria in the girder two key sections is insufficient for structural safety.

Fig. 4 – Comparison of the required prestressing force $P_m$ for internal and external prestressing for $\beta = 0.80$.

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Fig. 5 – Evaluation of the adequacy of serviceability reinforcement ($A_p$ and $A_{s,min}$) for the girders structural safety in flexure ($\Delta \sigma = 100$ Mpa for external prestressing).
Fig. 5 shows that the serviceability reinforcement is generally sufficient to satisfy structural safety requirements for the internally prestressed girders in the span range considered here. The externally prestressed girders however, require additional reinforcement, in particular in the higher span range. This additional requirement could consist of passive longitudinal reinforcement or additional prestressing, or a combination of both. This difference between both types of prestressing is linked to the assumed value of the prestressing cable stress at ultimate limit state. For internal prestressing, the flexural capacity of the section was calculated with the cable yield stress. For external prestressing, the assumed value of the cable stress calculation was performed with the long-term value of the cable tension increased by a stress increment (Δσ = 100 Mpa) to account for the elongation of the external cables at the ultimate limit state [6, 7].

3.3 Shear Reinforcement

Fig. 6 shows the comparison of the shear reinforcement in the support area of the middle span for internal and external prestressing (λ/h = 22). At constant slenderness, the required amount of reinforcement decreases as the span increases for both types of prestressing. This is due to the parallel increase of the prestressing force and of its contribution to the girder shear resistance. Fig. 6 also shows that the externally prestressed girder requires between 10 % and 15 % more shear reinforcement than the internally prestressed girder. This results from a combination of factors, including the smaller inclination of the external prestressing cables and the thinner webs of the externally prestressed section.

4. Conclusions

This comparative study was conducted to quantify the influence of the type of prestressing (internal or external) used in the design of a concrete box-girder highway bridge. For the girder configuration and under the simplifying assumptions used in this study, the following applies:

- The difference in the amount of prestressing between comparable internal and externally prestressed girders depends on the girder height. For example, external prestressing requires approximately 20 % more prestressing force for a girder height of 2.2 m. At girder heights above 3.5 m however, internal prestressing becomes the more favorable type of prestressing.

- Unlike internally prestressed girders, the ultimate flexural capacity of externally prestressed girders provided by the passive and prestressed reinforcement designed on the
basis of commonly used serviceability criteria for bridges (deflection and crack control) tends to be insufficient for structural safety.

- The required web shear reinforcement at the support is 10% to 15% higher in the case of external prestressing.

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References


