Fracture Mechanics for Structural Concrete

1. Introduction

2. Why do we need to consider fracture mechanics in concrete?

3. Fracture mechanics for concrete

4. Computational Models for fracture
What is fracture mechanics?

- When a crack length reaches a certain critical length, it can propagate, even though the average stress is much less than the tensile strength of the test specimen.

- Fracture mechanics tries to find the quantitative relations between crack length, material’s resistance to crack growth, and the stress at which cracks start to propagate.
Strength based approach

- With classical linear elasticity, we can calculate the stress concentration effect.
- However, at the very sharp crack tip, the theory generates an infinite stress.
- Evidently, it is nonphysical.
- If it is true, the strength will be near zero. Therefore, more rational methods are needed.
In this approach, the crack forms when total energy does not change or decrease, even though work is required for the crack to grow.

\[ \frac{dE}{dA} = \frac{d\Pi}{dA} + \frac{dW}{dA} = 0 \]

Critical condition

dA: Incremental crack area
E : Total energy
\( \Pi \) : Potential energy supplied by release in internal strain energy
W : Work required to create new surfaces
Energy-balance approach (cont’d)

\[
-\frac{d\Pi}{dA} = \frac{\pi \sigma^2 a}{E} \quad \text{and} \quad \frac{dW}{dA} = 2\gamma_s
\]

\(d\Pi/dA\): internal strain energy decreasing rate

\(dW/dA\): work required to create new surface \(dA\)

\(\gamma_s\): surface energy
Internal strain energy is released by creating free surface.

Energy-balance concept can be expressed in energy-release rate ($G$) concept.

Crack occurs when $G$ reaches a critical value.

\[
G = - \frac{d\Pi}{dA}
\]

$\Pi$ : Potential energy defined by $U-F$
$U$ : Strain energy stored in the body
$F$ : Work done by external force
Crack modes

Note that there is no compression crushing.
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Formation of cracks require a certain amount of energy, which agrees with the fracture mechanics concept.

Finite element analysis based on the strength criteria is unobjective around sharp crack, that is, the analysis depends on the choice of mesh size.
Limit analysis approach based on plasticity theory cannot be justified in brittle types of failures, e.g., punching shear, shear failure without shear reinforcement, etc.

Size effect can be addressed by fracture mechanics.

After all, tension softening type failure (Quasi-brittle failure) can be tackled by fracture mechanics.
Why fracture mechanics in concrete?

Mesh size dependence

Element size along crack (a) 50 (b) 25 (c) 5
Mesh size dependence (cont’d)

Why fracture mechanics in concrete?

Model (a) is more safe than model (c)?

Model (c) reflect the reality more accurately?
Why fracture mechanics in concrete?

Limit of limit analysis

Limit analysis can not explain crack propagation.
In classical linear elasticity, critical stress does not depend on the structure size.

However, plain concrete shows strong size effect, that is, the cracking stress depends on the specimen or structure size.
Bazant’s size effect relation

\[
\bar{\sigma}_N = \frac{f'_t}{\sqrt{1 + \frac{d}{\lambda d_a}}}
\]

- \(\bar{\sigma}_N\): prediction for the nominal stress
- \(f'_t\): tensile strength
- \(d_a\): maximum aggregate size
- \(\lambda\): an empirical constant
- \(d\): structure or specimen size
Double punch test

Why fracture mechanics in concrete?

Double Punch Test Setup

Test Result
Large Beam Failing in Shear

(Photo Courtesy of Shimizu Corporation)
Influence of Member Depth and Maximum Aggregate Size on Shear Stress at Failure (Experimental Results, Shioya)

\[ \frac{V}{b_w d} \] (MPa)

- PCA Tests of Model Air Force Warehouse Beams
  - max. aggregate size \( a = 10 \text{ mm} \)
  - \( a = 25 \text{ mm} \)
  - \( a = 2.5 \text{ mm} \)

ACI
- \( f_c' = 3500 \text{ psi (24.1 MPa)} \)
- \( f_{\text{yield}} = 52 \text{ ksi (386 MPa)} \)

MCFT \( a = 25 \text{ mm} \)
- \( b = 1500 \text{ mm} \)
- \( b = 1000 \text{ mm} \)
- \( b = 500 \text{ mm} \)

Air Force Warehouse Beams
- \( d = 100 \text{ mm} \)
- \( d = 200 \text{ mm} \)
- \( d = 600 \text{ mm} \)
- \( d = 1000 \text{ mm} \)
- \( d = 2000 \text{ mm} \)
- \( d = 3000 \text{ mm} \)

\[ \rho = \frac{A_s}{b_w d} = 0.004 \]
Shear Design of Transfer Beams

DL = 1650 kips
LL = 550 kips
Total = 2200 kips, service load

Design with Stirrups

$V_u = 1540$ kips  $f'_c = 5000$ psi  $a = 3/4''$
$V_n = 2240$ kips  $f_y = 60$ ksi  $\rho_{Long} = 1.90\%$

Alternative Design without Stirrups

$V_u = 1620$ kips  $f'_c = 10000$ psi  $a = 3/4''$
$V_n = 2210$ kips  $f_y = 60$ ksi  $\rho_{Long} = 0.93\%$
Construction and Loading of the Large Wide Beam, AT-1 Under Testing Machine at the University of Toronto
Observed Load-Deformation Response of Beam AT-1

ACI 318-02 Predicted failure load, $P = 1070$ kips

ACI 318-02 Predicted service load, $P = 600$ kips

Observed failure load
$P = 549$ kips

$\frac{\Delta}{L} = \frac{1}{570}$
Crack Development for Beam AT-1
Failure of Large Beam at 47% of ACI Shear Failure Load
Observed Load-Deformation Response of Beam AT-1

\[
\frac{V_c}{b_w \sqrt{f'_c}}
\]

If \( f'_c > 10000 \text{ psi} \),

- take \( \sqrt{f'_c} = 100 \text{ psi} \) and \( a = 0 \)

ACI 318-02

\[
V_c = 2 \sqrt{f'_c} b_w d
\]

MCFT

\[
V_c = \frac{100}{38 + \sqrt{f'_c}} b_w d
\]

\[
s_e = \frac{1.24 d}{a + 0.63}
\]

- Experiments \( a \leq 0.2 \text{ inch or } f'_c > 9000 \text{ psi} \)

- Bahen Center alternate design
Influence of Minimum Shear Reinforcement on Load-Deformation Response of Large Beams

$P = 292$ kips

$\Delta = 3.3$ in.

$L = 35' - 5''$

$f_c' = 5100$ psi

$A_s = 6.51$ in$^2$

$a = 3/8$ in.

$f_y = 66.3$ ksi

6 - No. 30M bars

$\rho = 0.74\%$

$\frac{\Delta}{L} = \frac{1}{129}$

$A_s f_y = 50$ psi

$P = 292$ kips

$\frac{A_s f_y}{b_w s} = 0$

$P = 104$ kips

$\frac{\Delta}{L} = \frac{1}{1350}$

1 leg of #4 @ 23 in.

$f_y = 67.9$ ksi

74.4''

11.8''

YB2000/0

YB2000/4
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Linear elastic fracture mechanics allows the stress to approach infinity at a crack tip. However, infinite stress cannot develop in real materials. A certain range of inelastic zone must exist around crack tip. In metal, this zone is a yielding zone.
- Includes many microcracks.
- Cohesive pressure still exists.
- Must be nonlinear.
- Finite tensile strength $f'$.
- Size is much larger than steel.
- In contrast to metals, strain-softening, instead of strain-hardening, dominates.
Comparison of crack tips

Fracture mechanics for concrete

Linear Fracture  Metal  Concrete

F : Fracture process zone
N : Nonlinear hardening zone
L : Linear zone
Comparison of crack tips (Cont’d)

- In ductile metal, fracture process zone, which shows strain-softening, is usually small.
- In concrete, fracture process zone is large compared to metal.
- Therefore, in concrete, crack tip is not defined clearly.
Fracture energy

- Fracture energy is the energy required to open unit area of crack surface.
- Fracture energy is a material constant.
- It is independent of structure size or geometry of structure.
Fracture mechanics for concrete

Fracture energy (Cont’d)

\[ G_q = G_{Ic} + \int_{0}^{w_t} \sigma(w)dw \]

\( G_q \): Fracture energy

\( G_{Ic} \): Energy to create new surface

\( \int_{0}^{w_t} \sigma(w)dw \): Energy to separate the surfaces completely
In general, $G_{ic}$ is ignored.

To determine this curve, $f'$, $G_f$, and the shape of curve have to be known.

The shape can be linear, bi-linear, quadratic, etc.

Note that the energy is not a function of strain, but displacement.
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Two models

- Fictitious Crack Model by Hillerborg (1976)
  - First proposed model applying fracture energy
  - Discrete model approach

- Crack Band Model by Bazant and Oh (1983)
  - Smeared crack model approach
  - Proposed minimum element size
Before the Hillerborg model...

- In the Dugdale model, plastic zone near the crack tip.
- In that model, the stress equals to the yield stress across the crack.
- Barenblatt model is similar to the Dugdale model, but the stress is assumed to vary with the deformation.
Computational Models for Fracture

**Fictitious crack model - assumption**

- The crack is assumed to propagate when the stress reaches the tensile strength, $f'$.  
- When the crack opens, the stress is not assumed to fall to zero at once.  
- After cracking, the stress at the node is calculated by the fracture energy curve.  
- When implemented in finite element method, the node is separated into two nodes. (discrete model)
Fictitious crack model - Example

Three points bend beam

Finite element mesh

F1 is computed based on fracture energy, not by the linear elastic fracture mechanics

Result
Characteristic length

- Hillerborg introduced the concept of characteristic length, \( l_c = EG_c / f_i^{1/2} \)

- Characteristic length is a material property also.

- Characteristic length is an index for the brittleness.

- If it is large, the material is ductile, whereas if it is small, brittle.
Fictitious crack mode - Shortcomings

- When the crack extends through a certain node, the node must be split into two nodes.
- The optimized band should be calculated in each step.
- If the direction of crack is not known in advance, identifying the direction in which the energy release rate is maximum should be carried out.
Fictitious crack mode – Shortcomings (cont’d)

- Even if the calculation is possible, the direction may not propagate through nodes. In that case, the calculation may be misleading.
Crack band model - Introduction

- In the theory of randomly inhomogeneous materials, equivalent continuum stresses and strains are defined over a certain representative volume.
- The volume size must be at least several times the maximum aggregate size.
- The distribution of stress or strain over distance less than several aggregate sizes has no physical meaning.
Bazant and Oh modeled the fracture process zone by a band of *smeared crack* band with a fixed width $h_c$.

To remedy the unobjectivity of finite element analysis, cracking criterion of fracture energy was introduced.
Crack width of crack band model

- In the smeared crack band, the crack opening, \( w \), is calculated as the product of average strain and band width.

- Note that the fracture energy relation is changed into one between stress and strain.
Crack band width

- In those paper, Bazant and Oh assumed stress strain relation in the fracture process zone as linear.
- In this case, the crack band width ($w_c$) is calculated as

$$w_c = \frac{2G_f}{f_t^{1/2}} \left( \frac{1}{E} - \frac{1}{E_t} \right)^{-1}$$
Characteristics of cracked element

If a certain element is in fracture process zone, the stiffness in the direction normal to crack is decreased gradually according to fracture energy relationship.
Characteristics of cracked element (Cont’d)

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\end{bmatrix} = \frac{1}{E} \begin{bmatrix}
1 & -\nu & -\nu \\
-\nu & 1 & -\nu \\
-\nu & -\nu & 1 \\
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
\varepsilon_f \\
\end{bmatrix}
\]

Where x, y and z are principal direction.

\( \varepsilon_f \) is the additional strain due to opening of the microcracks.

\[
\varepsilon_f = \frac{1}{C_f} \left( f'_i - \sigma_z \right)
\]
Characteristics of cracked element
(Cont’d)

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & \frac{E}{E_t} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \varepsilon_0 \end{bmatrix}$$

where

$$\frac{1}{E_t} = \frac{1}{E} - \frac{1}{C_f} \leq 0$$

$$\varepsilon_0 = \frac{f'_t}{C_f}$$

Finally, for plane stress case, orthotropic stiffness matrix is

$$\begin{bmatrix} \sigma_x \\ \sigma_y \end{bmatrix} = \begin{bmatrix} E + \nu^2 E'_t & \nu E'_t \\ \nu E'_t & E'_t \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_z - \varepsilon_0 \end{bmatrix}$$

where

$$E'_t = \frac{1}{\frac{1}{E'_t} - \frac{1}{C_f}}$$

$$E' = \frac{E}{1 - \nu^2}$$
Element size of crack band model

- In finite element analysis, the crack band width is the element size of fracture process path.
- Since the size of fracture process zone is $w_c$, the smaller element than $w_c$ makes no sense.
- However, larger element than $w_c$ can be used with proper correction.
- The crack width will be the product of crack band width and element strain.