fib Model Code 2010



# *Punching of flat slabs: Design example*



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#### 1.1 Geometry (dimensions in [m])





0.25

flat slab

column



Spans:  $L_x = 6.00 \text{ m}$  and  $L_y = 5.60 \text{ m}$ 

The material properties can be found in chapter 5 of model code 2010.





Concrete C30		Steel B500S (flexural and transverse reinforcement)		
$f_{ck}$	30 MPa	$f_{yd}$	435 MPa	
Ус	1.5	$E_s$	200 GPa	
$d_g$	32 mm	Ductility class	В	

#### Loads 1.3

Self-weight of concrete slab:	6.25 kN/m <sup>2</sup>
Superimposed dead load:	$2 \text{ kN/m}^2$
Live load:	3 kN/m <sup>2</sup>

 $g_d + q_d = 1.35(6.25 + 2) + 1.5 \cdot 3 = 15.6 \text{ kN/m}^2$ 



## The goal of the preliminary design is to check if the dimensions of the structure are reasonable with respect to the punching shear strength and if punching shear reinforcement is needed.

The reaction forces in the columns are estimated by using contributive areas.

The effective depth  $d_v$  is assumed to be 200 mm.

Eccentricity coefficient ( $k_e$ ) are adopted from the commentary of §7.3.5.2



According to the commentary, the distance to the point where the radial moment is zero  $r_s$  can be estimated based on the spans.

By using the Level I approach, one can estimate the rotations.

The maximum aggregate size of 32 mm leads to a factor  $k_{dg}$  of

$$k_{dg} = \frac{32}{16 + d_g} = \frac{32}{16 + 32} = 0.67 < 0.75 \rightarrow k_{dg} = 0.75$$

#### 2 Level I of approximation (preliminary design)

Reaction forces



To check if shear reinforcement and which system can be used, one can calculate the minimal needed value of factor  $k_{sys}$ .

$$V_{Rd,\max} = k_{sys}k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} \ge V_d \rightarrow k_{sys} \ge \frac{V_d}{k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi}} = \frac{V_d}{V_{Rd,c}}$$

 $k_{sys}$  depends on the performance of the used shear reinforcement system. The model code proposes a value of  $k_{sys} = 2.0$  for system compliant with model code detailing rules (§7.13.5.3). Higher values (up to  $k_{sys} = 2.8$ ) can be used if more restrictive detailing rules are adopted and if the placing of the transverse reinforcement is checked at the construction site.

Shear strength without shear reinforcement

Inner: 
$$V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} = 0.227 \frac{\sqrt{30}}{1.5} 1501 \cdot 200 \cdot 10^{-3} = 249 \text{ kN} \le V_d = 692 \text{ kN}$$
  
Corner:  $V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} = 0.227 \frac{\sqrt{30}}{1.5} 440 \cdot 200 \cdot 10^{-3} = 73 \text{ kN} \le V_d = 93 \text{ kN}$   
Edge:  $V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} = 0.227 \frac{\sqrt{30}}{1.5} 766 \cdot 200 \cdot 10^{-3} = 127 \text{ kN} \le V_d = 265 \text{ kN}$ 

The thickness of the slab has to be increased or the slab has to be shear reinforced.

Shear reinforcement

Inner: 
$$k_{sys} \ge \frac{V_d}{V_{Rd,c}} = \frac{692}{249} = 2.78$$

Corner:  $k_{sys} \ge \frac{V_d}{V_{Rd_c}} = \frac{93}{73} = 1.27$ 

Edge: 
$$k_{sys} \ge \frac{V_d}{V_{Rd,c}} = \frac{265}{127} = 2.09$$

#### Conclusions

Inner column: Shear reinforcement is necessary and sufficient (accounting for the values of  $k_{sys}$ ) to ensure punching shear strength

Corner columns: Shear reinforcement might probably not be necessary. This has to be confirmed by a higher level of approximation.

Edge columns: Shear reinforcement might probably be necessary.

The thickness of the slab is sufficient if shear reinforcement is used.

#### 3 Level II of approximation (typical design)

The moments and the reaction forces have been calculated with a finite element software. For the analysis, a linear-elastic model has been used.

The moment  $M_d$  is the vector addition of the moments in x- and y-direction.

 $M_{d} = \sqrt{M_{d,x}^{2} + M_{d,y}^{2}}$ 

For a level II approximation, one has to know the flexural reinforcement. It was designed on the basis of the previous finite element analysis.

#### 3.4 Structural analysis and flexural design

Summary of the column reactions



The flexural strength can be calculated according to the Model Code. In this example, however, the flexural strength has been calculated assuming a rigid-plastic behavior of concrete and steel:

$$m_{Rd} = \rho \cdot d^2 \cdot f_{yd} \left( 1 - \frac{\rho \cdot f_{yd}}{2f_{cd}} \right)$$

The design shear force  $V_d$  is equal to the column reaction force  $N_d$  minus the applied load within the control perimeter  $(g_d + q_d) \cdot A_c$ .

_	<i>d</i> <sub>v</sub> /, ▼	2 <u> </u>	$\frac{d_v/2}{g_d}$	$+q_d$
	<b>v</b> <sub>d</sub>		Vd	$d_v$

In case of inner columns, the centroid of the column corresponds to the centroid of the control perimeter. Therefore,  $-\frac{d_v/2}{b_c} \frac{b_c}{d_v/2}$ 

 $\Delta e = 0$ 

The distances  $r_{s,x}$  and  $r_{s,y}$  are the same as for the Level I approximation.

 $k_{dg}$  is calculated at Level I.

#### 3.5 Shear design inner column C5

Design shear force

$$A_{c} = b_{c}^{2} + 2 \cdot b_{c} \cdot d_{v} + \frac{d_{v}^{2}}{4}\pi = 0.26^{2} + 2 \cdot 0.26 \cdot 0.204 + \frac{0.204^{2}}{4}\pi = 0.21m^{2}$$
$$V_{d} = N_{d} - (g_{d} + q_{d}) \cdot A_{c} = 664 - 15.6 \cdot 0.21 = 661 \text{ kN}$$

Control perimeter

$$e_{u} = \left| \frac{M_{d}}{V_{d}} - \Delta e \right| = \left| \frac{8 \cdot 10^{6}}{661 \cdot 10^{3}} - 0 \right| = 12 \text{ mm} \qquad b_{u} = \sqrt{\frac{4}{\pi} A_{c}} = \sqrt{\frac{4}{\pi} 206 \cdot 10^{3}} = 513 \text{ mm}$$

$$k_{e} = \frac{1}{1 + e_{u}/b_{u}} = \frac{1}{1 + 12/513} = 0.98$$

$$b_{0} = k_{e} \cdot b_{1} = k_{e} \cdot (4 \cdot b_{c} + d_{v} \cdot \pi) = 0.98 \cdot (4 \cdot 260 + 204 \cdot \pi) = 1642 \text{ mm}$$

Rotations

$$\begin{aligned} r_{s,x} &= 1.32 \,\mathrm{m} \qquad r_{s,y} = 1.23 \,\mathrm{m} \\ b_s &= 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \,\mathrm{m} \\ m_{sd,x} &= \frac{V_d}{8} + \left| \frac{M_{d,x} - V_d \cdot \Delta e_x}{2 \cdot b_s} \right| = \frac{661}{8} + \left| \frac{8}{2 \cdot 1.91} \right| = 85 \,\mathrm{kNm/m} \\ m_{sd,y} &= \frac{V_d}{8} + \left| \frac{M_{d,y} - V_d \cdot \Delta e_y}{2 \cdot b_s} \right| = \frac{661}{8} + \left| \frac{1}{2 \cdot 1.91} \right| = 83 \,\mathrm{kNm/m} \\ \psi_x &= 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \cdot \left( \frac{m_{sd}}{m_{Rd,x}} \right)^{1.5} = 1.5 \cdot \frac{1.32}{0.204} \frac{435}{200000} \cdot \left( \frac{85}{115} \right)^{1.5} = 0.0133 \, \Leftrightarrow \,\mathrm{governing} \\ \psi_y &= 1.5 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_s} \cdot \left( \frac{m_{sd}}{m_{Rd,y}} \right)^{1.5} = 1.5 \cdot \frac{1.23}{0.204} \frac{435}{200000} \cdot \left( \frac{83}{115} \right)^{1.5} = 0.0121 \\ k_\psi &= \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.9 \cdot 0.0133 \cdot 204 \cdot 0.75} = 0.300 \le 0.6 \end{aligned}$$

The punching shear strength of the concrete is not sufficient. Consequently, shear reinforcement is necessary.

Firstly, one has to check if the design shear force  $V_d$  is smaller than the maximum punching strength  $V_{Rd,max}$ . This is done assuming  $k_{sys}=2$ .

The design shear force  $V_d$  is below the maximum punching strength  $V_{Rd,max}$ . Therefore, the slab can be reinforced with shear reinforcement complying with detailing rules defined in subclause §7.13.5.3.

The bond strength is taken as  $f_{bd} = 3$  MPa (according to MC 2010 for corrugated bars).

To avoid a failure outside the shear reinforced area, the outer perimeter need to have a minimal length. The design shear force can be reduced to account for the loads applied inside the outer perimeter. This effect is neglected as a safe estimate.

In this example, the calculating value of the effective depth  $d_v$  is equal to the effective depth *d* minus the concrete cover *c* on the bottom surface of the slab:  $d_{v,out} = d - c = 204 - 30 = 174 \, mm$ 

Assuming a circular control perimeter for the estimation of the eccentricity, factor  $k_e$  can be estimated as detailed in the right hand side column.

Possible shear reinforcement layout:





Punching strength without shear reinforcement

$$V_{_{Rd,c}} = k_{_{\psi}} \frac{\sqrt{f_{_{ck}}}}{\gamma_{_c}} b_{_0} d_{_{\psi}} = 0.300 \frac{\sqrt{30}}{1.5} 1642 \cdot 204 \cdot 10^{_{-3}} = 367 \text{ kN} \le V_{_d} = 661 \text{ kN}$$

Punching strength with shear reinforcement

$$\begin{aligned} V_{Rd,max} &= k_{sy} V_{Rd,c} = 2 \cdot 367 = 734 \text{ kN} \le \frac{\sqrt{f_{cd}}}{\gamma_c} b_0 d_v = 1223 \text{ kN} \\ V_{Rd,max} &= 734 \text{ kN} \ge V_d = 661 \text{ kN} \\ \sigma_{sved} &= \frac{E_s \psi}{6} \cdot \left(1 + \frac{f_{bd}}{f_{yvd}} \cdot \frac{d}{\phi_v}\right) = \frac{200000 \cdot 0.0133}{6} \cdot \left(1 + \frac{3}{435} \cdot \frac{204}{8}\right) \\ \sigma_{sved} &= 521 \text{ MPa} > f_{yvd} = 435 \text{ MPa} \\ A_{sw} &= \frac{V_d - V_{Rd,c}}{k_c \sigma_{sved}} \sin \alpha} = \frac{(661 - 367)}{0.98 \cdot 435 \cdot \sin(90^\circ)} = 690 \text{ mm}^2 \\ A_{swe,min} &= \frac{0.5 \cdot V_d}{k_e \sigma_{sved}} \sin \alpha} = \frac{0.5 \cdot 661}{0.98 \cdot 435 \cdot \sin(90^\circ)} = 779 \text{ mm}^2 \Leftrightarrow \text{ governing} \\ b_0 &= \frac{V_d}{k_v \sqrt{\frac{f_{ck}}{\gamma_c}}} d_{v,out}} = \frac{661 \cdot 10^3}{0.300 \frac{\sqrt{30}}{1.5}} 174 \\ r_{out} &= \frac{b_0}{2\pi} = \frac{3468}{2\pi} = 552 \text{ mm} \qquad k_e = \frac{1}{1 + e_u/2r_{out}} = \frac{1}{1 + 12/(2 \cdot 552)} = 0.99 \\ b_{out} &= \frac{b_0}{k_e} = \frac{3468}{0.99} = 3505 \text{ mm} \\ \rho_{sw} &= \varphi_w \left[ 4 \cdot b_c \cdot d_v + d_v^2 \cdot \pi - 4 \cdot b_c \cdot 0.35 d_v - (0.35 d_v)^2 \cdot \pi \right] \\ A_{ov} &= 1263 \text{ mm}^2 > 774 \text{ mm}^2 \\ b_{out} &= 4 \cdot 800 + 174 \cdot \pi = 3746 \text{ mm} > 3505 \text{ mm} \end{aligned}$$

The design shear force  $V_d$  is equal to the column reaction force  $N_d$  minus the applied load within the control perimeter  $(g_d + q_d) \cdot A_c$ .

$$\Delta e_{x} = \Delta e_{y} = \frac{3}{4} \left( b_{c} + \frac{d_{v}}{2} \right) - \frac{b_{c}}{2} = \frac{1}{4} \left( b_{c} + \frac{3}{2} d_{v} \right)$$





#### 3.6 Shear design corner column C1 and C3

Design shear force

$$A_{c} = b_{c}^{2} + 2 \cdot b_{c} \cdot \frac{d_{v}}{2} + \frac{d_{v}^{2}}{16}\pi = 0.26^{2} + 0.26 \cdot 0.210 + \frac{0.210^{2}}{16}\pi = 0.13 m^{2}$$
$$V_{d} = N_{d} - (g_{d} + q_{d}) \cdot A_{c} = 112 - 15.6 \cdot 0.13 = 110 \text{ kN}$$

Control perimeter

$$\Delta e_x = \Delta e_y = \frac{1}{4} \left( b_c + \frac{3}{2} d_y \right) = \frac{1}{4} \left( 260 + \frac{3}{2} 210 \right) = 144 \text{ mm}$$

$$\Delta e = \Delta e_x \cdot \sqrt{2} = 144 \cdot \sqrt{2} = 203 \text{ mm}$$

$$e_u = \left| \frac{M_d}{V_d} - \Delta e \right| = \left| \frac{33 \cdot 10^6}{110 \cdot 10^3} - 203 \right| = 97 \text{ mm} \qquad b_u = \sqrt{\frac{4}{\pi}} A_c = \sqrt{\frac{4}{\pi}} 131 \cdot 10^3 = 408 \text{ mm}$$

$$k_e = \frac{1}{1 + e_u / b_u} = \frac{1}{1 + 97/408} = 0.81$$

$$b_0 = k_e \cdot b_1 = k_e \cdot \left( 2 \cdot b_c + \frac{d_v \cdot \pi}{4} \right) = 0.81 \cdot \left( 2 \cdot 260 + 210 \cdot \frac{\pi}{4} \right) = 554 \text{ mm}$$

Rotations

$$\begin{aligned} r_{s,x} &= 1.32 \,\mathrm{m} & r_{s,y} = 1.23 \,\mathrm{m} \\ b_s &= 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \,\mathrm{m} \\ b_{sr} &= 2 \cdot b_c = 2 \cdot 0.26 = 0.52 \,\mathrm{m} \Rightarrow \text{governing} \\ m_{sd,x} &= \frac{V_d}{8} + \left| \frac{M_{d,x} - V_d \cdot \Delta e_x}{b_s} \right| = \frac{110}{8} + \left| \frac{25 - 110 \cdot 0.144}{0.52} \right| = 31 \,\mathrm{kN} < \frac{V_d}{2} = \frac{110}{2} = 55 \,\mathrm{kN} \\ m_{sd,y} &= \frac{V_d}{8} + \left| \frac{M_{d,y} - V_d \cdot \Delta e_y}{b_s} \right| = \frac{110}{8} + \left| \frac{22 - 110 \cdot 0.144}{0.52} \right| = 25 \,\mathrm{kN} < \frac{V_d}{2} = \frac{110}{2} = 55 \,\mathrm{kN} \\ \psi_x &= 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \cdot \left( \frac{m_{sd,x}}{m_{Rd,x}} \right)^{1.5} = 1.5 \cdot \frac{1.32}{0.21} \frac{435}{200000} \cdot \left( \frac{55}{69} \right)^{1.5} = 0.0146 \, \Rightarrow \,\mathrm{governing} \end{aligned}$$

The distances  $r_{s,x}$  and  $r_{s,y}$  are the same as for the Level I approximation.

In case of corner columns, the width of the support strip may be limited by the distance  $b_{sr}$ .



 $k_{dg}$  is calculated at Level I.

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary

Since no shear reinforcement has been used and  $m_{sd} < m_{Rd}$ , integrity reinforcement needs to be provided.

For the design of the integrity reinforcement, the accidental load case can be used. Thus, the design load can be reduced.

$$(g_d + q_d)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1 \text{ kN/m}^2$$
$$V_{d,acc} = \frac{(g_d + q_d)_{acc}}{g_d + q_d} \cdot V_d = \frac{10.1}{15.6} \cdot 110 = 71 \text{ kN}$$

The material properties can be found in chapter 5 of model code 2010.

Ductility class B :  $(f_t/f_y)_k = 1.08$ 

It is assumed that only straight bars will be used, thus  $\alpha_{ult} = 20^\circ$ .

With respect to integrity reinforcement, two restrictions should be fulfilled: -the integrity reinforcement should at least be composed of four bars -the diameter of the integrity bars  $\phi_{int}$  has to be chosen such that

-the diameter of the integrity bars  $\phi_{int}$  has to be chosen such that  $\phi_{int} \leq 0.12 d_{res}$ 



×.	$l_b$	

$$\psi_{y} = 1.5 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_{s}} \cdot \left(\frac{m_{sd,y}}{m_{Rd,y}}\right)^{1.5} = 1.5 \cdot \frac{1.23}{0.21} \frac{435}{200000} \cdot \left(\frac{55}{69}\right)^{1.5} = 0.0136$$
$$k_{\psi} = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dy}} = \frac{1}{1.5 + 0.9 \cdot 0.0146 \cdot 210 \cdot 0.75} = 0.280 \le 0.6$$

Punching strength without shear reinforcement

$$V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} = 0.280 \frac{\sqrt{30}}{1.5} 554 \cdot 210 \cdot 10^{-3} = 119 \text{ kN} > V_d = 110 \text{ kN}$$

Integrity reinforcement

$$A_{s} = \frac{V_{d,acc}}{f_{yd} \cdot (f_{t}/f_{y})_{k} \cdot \sin \alpha_{ult}} = \frac{71 \cdot 10^{3}}{435 \cdot 1.08 \cdot \sin(20^{\circ})} = 442 \,\mathrm{mm}^{2}$$
$$V_{d,acc} \le \frac{\sqrt{f_{ck}}}{\gamma_{c}} b_{int} d_{res} = \frac{\sqrt{30}}{1.5} \cdot 464 \cdot 168 = 285 \,\mathrm{kN}$$

 $\Rightarrow$  2x2 ø12  $A_s$  = 452 mm<sup>2</sup> (2 in each direction with a spacing of 100 mm)

$$d_{res} = h - 2 \cdot c - \phi_{top} - \phi_{bottom} = 250 - 2 \cdot 30 - 10 - 12 = 168 \text{ mm}$$
$$b_{int} = 2 \cdot s_{int} + \frac{\pi}{2} d_{res} = 2 \cdot 100 + \frac{\pi}{2} \cdot 168 = 464 \text{ mm}$$
$$\phi_{int} = 12 \text{ mm} \le 0.12 d_{res} = 0.12 \cdot 168 = 20 \text{ mm}$$

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The design shear force  $V_d$  is equal to the column reaction force  $N_d$  minus the applied load within the control perimeter  $(g_d + q_d) \cdot A_c$ .

$$\Delta e_{x} = \frac{\left(b_{c} + d_{v}\right) \cdot \left(b_{c} + \frac{d_{v}}{2}\right) + 2\left(b_{c} + \frac{d_{v}}{2}\right) \cdot \frac{1}{2}\left(b_{c} + \frac{d_{v}}{2}\right)}{\left(b_{c} + d_{v}\right) + 2\left(b_{c} + \frac{d_{v}}{2}\right)} - \frac{b_{c}}{2}$$

$$\Delta e_{x} = \frac{1}{4} \cdot \frac{2b_{c}^{2} + 6b_{c}d_{v} + 3d_{v}^{2}}{3b_{c} + 2d_{v}}$$

The distances  $r_{s,x}$  and  $r_{s,y}$  are the same as for the Level I approximation.

In case of edge columns, the width of the support strip may be limited by the distance  $b_{sr}$ .



#### 3.7 Shear design edge column C2

#### Design shear force

$$A_{c} = b_{c}^{2} + 3 \cdot b_{c} \cdot \frac{d_{v}}{2} + \frac{d_{v}^{2}}{8}\pi = 0.26^{2} + \frac{3}{2}0.26 \cdot 0.21 + \frac{0.21^{2}}{8}\pi = 0.17 \,\mathrm{m}^{2}$$
$$V_{d} = N_{d} - (g_{d} + q_{d}) \cdot A_{c} = 266 - 15.6 \cdot 0.17 = 263 \,\mathrm{kN}$$

#### Control perimeter

$$\Delta e_x = \frac{1}{4} \cdot \frac{2b_c^2 + 6b_c d_v + 3d_v^2}{3b_c + 2d_v} = \frac{1}{4} \cdot \frac{2 \cdot 260^2 + 6 \cdot 260 \cdot 210 + 3 \cdot 210^2}{3 \cdot 260 + 2 \cdot 210} = 124 \text{ mm}$$
  

$$\Delta e_y = 0 \text{ mm} \qquad \Delta e = \Delta e_x = 124 \text{ mm}$$
  

$$e_u = \left| \frac{M_d}{V_d} - \Delta e \right| = \left| \frac{42 \cdot 10^6}{263 \cdot 10^3} - 124 \right| = 35 \text{ mm} \qquad b_u = \sqrt{\frac{4}{\pi}} A_c = \sqrt{\frac{4}{\pi}} \frac{167 \cdot 10^3}{167 \cdot 10^3} = 461 \text{ mm}$$
  

$$k_e = \frac{1}{1 + e_u / b_u} = \frac{1}{1 + 35/461} = 0.93$$
  

$$b_0 = k_e \cdot b_1 = k_e \cdot \left( 3 \cdot b_c + \frac{d_v \cdot \pi}{2} \right) = 0.93 \cdot \left( 3 \cdot 260 + 210 \cdot \frac{\pi}{2} \right) = 1031 \text{ mm}$$

#### Rotations

$$r_{s,x} = 1.32 \text{ m} \qquad r_{s,y} = 1.23 \text{ m}$$

$$b_s = 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \text{ m}$$

$$b_{sr,x} = 3 \cdot b_c = 3 \cdot 0.26 = 0.78 \text{ m} \Rightarrow \text{governing}$$

$$b_{sr,y} = \frac{b_c}{2} + \frac{b_s}{2} = \frac{0.260}{2} + \frac{1.91}{2} = 1.09 \text{ m} \Rightarrow \text{governing}$$

$$m_{sd,x} = \frac{V_d}{8} + \left| \frac{M_{d,x} - V_d \cdot \Delta e_x}{b_s} \right| = \frac{263}{8} + \left| \frac{42 - 263 \cdot 0.124}{0.78} \right| = 45 \text{ kN}$$

$$m_{sd,y} = \frac{V_d}{8} + \left| \frac{M_{d,y} - V_d \cdot \Delta e_y}{2 \cdot b_s} \right| = \frac{263}{8} + \left| \frac{0}{2 \cdot 1.09} \right| = 33 \text{ kN} < \frac{V_d}{4} = \frac{263}{4} = 66 \text{ kN}$$

$$\psi_{x} = 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_{s}} \cdot \left(\frac{m_{sd,x}}{m_{Rd,x}}\right)^{1.5} = 1.5 \cdot \frac{1.32}{0.21} \frac{435}{200000} \cdot \left(\frac{45}{69}\right)^{1.5} = 0.011$$
$$\psi_{y} = 1.5 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_{s}} \cdot \left(\frac{m_{sd,y}}{m_{Rd,y}}\right)^{1.5} = 1.5 \cdot \frac{1.23}{0.21} \frac{435}{200000} \cdot \left(\frac{66}{69}\right)^{1.5} = 0.018 \Rightarrow \text{governing}$$
$$k_{\psi} = \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.9 \cdot 0.018 \cdot 210 \cdot 0.75} = 0.247 \le 0.6$$

Punching strength without shear reinforcement

$$V_{Rd,c} = k_{\psi} \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_{\psi} = 0.247 \frac{\sqrt{30}}{1.5} 1031 \cdot 210 \cdot 10^{-3} = 195 \text{ kN} < V_d = 263 \text{ kN}$$

 $k_{dg}$  is calculated at Level I.

The punching shear strength of the concrete is not sufficient. Since the strength seems to be rather close to the design load, a level III approximation will be performed.

The Level III calculations are based on the results of the linear-elastic finite element analysis.

From the results of the flexural analysis, one can obtain the distance between the center of the column and the point, at which the bending moments are zero.

The average moment in the support strip can be obtained by the integration of the moments at the strip section.

Since the flexural moments  $m_{d,x}$  and  $m_{d,y}$  at the support regions are negative, the absolute value of the twisting moment  $m_{d,x}$  need to be subtracted so that the absolute value of  $m_{sd,x}$  and  $m_{sd,y}$  will be maximized.

$$m_{sd,x} = m_{d,x} - \left| m_{d,xy} \right|$$
$$m_{sd,y} = m_{d,y} - \left| m_{d,xy} \right|$$

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary

Since no shear reinforcement has been used and  $m_{sd} < m_{Rd}$ , integrity reinforcement needs to be provided to prevent a progressive collapse of the structure.

For the design of the integrity reinforcement, the accidental load case can be used. Thus the design load can be reduced.

$$(g_d + q_d)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1 \text{ kN/m}^2$$
$$V_{d,acc} = \frac{(g_d + q_d)_{acc}}{g_d + q_d} \cdot V_d = \frac{10.1}{15.6} \cdot 263 = 171 \text{ kN}$$





### 4 Level III of approximation (detailed design or assessment of existing structure)

$$\begin{aligned} r_{s,x} &= 0.64 \text{ m} & r_{s,y} = 1.18 \text{ m} \\ b_s &= 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} = 1.5 \cdot \sqrt{0.64 \cdot 1.18} = 1.30 \text{ m} \\ b_{sr,x} &= 3 \cdot b_c = 3 \cdot 0.26 = 0.78 \text{ m} \\ b_{sr,y} &= \frac{b_c}{2} + \frac{b_z}{2} = \frac{0.26}{2} + \frac{1.30}{2} = 0.78 \text{ m} \\ m_{sd,x} &= 24 \text{ kNm/m} & m_{sd,y} = 43 \text{ kNm/m} \quad \text{(average value on support strip)} \\ r_{s,x} &= 0.64 \text{m} > \frac{2}{3} b_{sr,x} = 0.52 \text{ m} & r_{s,y} = 1.18 \text{ m} > \frac{2}{3} b_{sr,y} = 0.52 \text{ m} \\ \psi_x &= 1.2 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd}}{m_{Rd,y}}\right)^{15} = 1.2 \cdot \frac{0.64}{0.21} \frac{435}{200000} \cdot \left(\frac{24}{69}\right)^{15} = 0.0016 \\ \psi_y &= 1.2 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd}}{m_{Rd,y}}\right)^{15} = 1.2 \cdot \frac{1.18}{0.21} \frac{435}{200000} \cdot \left(\frac{43}{69}\right)^{15} = 0.0073 \Rightarrow \text{governing} \\ k_y &= \frac{1}{1.5 + 0.9 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.9 \cdot 0.0073 \cdot 210 \cdot 0.75} = 0.395 \le 0.6 \\ V_{Rd,s} &= k_y \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_y = 0.395 \frac{\sqrt{30}}{1.5} 1031 \cdot 210 \cdot 10^{-3} = 312 \text{ kN} > V_d = 263 \text{ kN} \end{aligned}$$

Integrity reinforcement

$$A_{s} = \frac{V_{d,acc}}{f_{yd} \cdot (f_{t}/f_{y})_{k} \cdot \sin \alpha_{ult}} = \frac{171 \cdot 10^{3}}{435 \cdot 1.08 \cdot \sin(20^{\circ})} = 1064 \text{ mm}^{2}$$
$$V_{d,accs} \le \frac{\sqrt{f_{ck}}}{\gamma_{c}} b_{int} d_{res} = \frac{\sqrt{30}}{1.5} \cdot 861 \cdot 166 = 522 \text{ kN}$$

$$\Rightarrow$$
 3x3 ø14  $A_s$  = 1385 mm<sup>2</sup> (3 in each direction with a spacing of 100 mm)

$$d_{res} = h - 2 \cdot c - \phi_{top} - \phi_{bottom} = 250 - 2 \cdot 30 - 10 - 14 = 166 \,\mathrm{mm}$$

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The material properties can be found in chapter 5 of model code 2010.

Ductility class B :  $(f_t/f_y)_k = 1.08$ 

It is assumed that only straight bars will be used, thus  $\alpha_{ult} = 20^{\circ}$ .

With respect to integrity reinforcement, two restrictions should be fulfilled: -the integrity reinforcement should at least be composed of four bars -the diameter of the integrity bars  $\phi_{int}$  has to be chosen such that  $\phi_{int} \leq 0.12 \cdot d_{res}$ 



	· 
$\sim$	$l_b$

Corners of walls should be checked following the same methodology.

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$$b_{\text{int}} = 3 \cdot s_{\text{int}} + \frac{\pi}{2} d_{\text{res}} = 3 \cdot 200 + \frac{\pi}{2} \cdot 166 = 861 \,\text{mm}$$
$$\phi_{\text{int}} = 14 \,\text{mm} \le 0.12 d_{\text{res}} = 0.12 \cdot 166 = 20 \,\text{mm}$$

