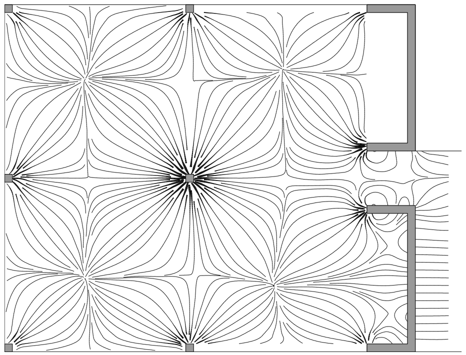

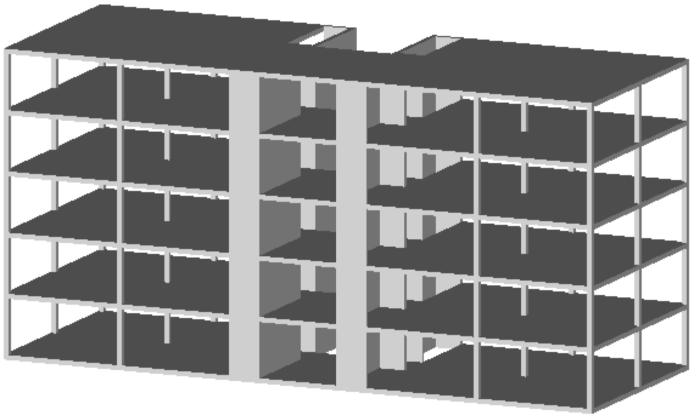

fib *Model Code 2010*

Punching of flat slabs: Design example

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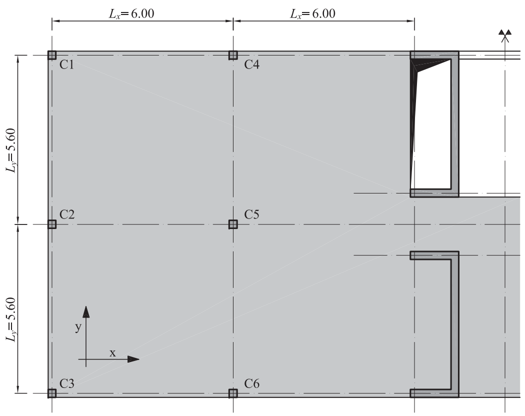
18.07.2011

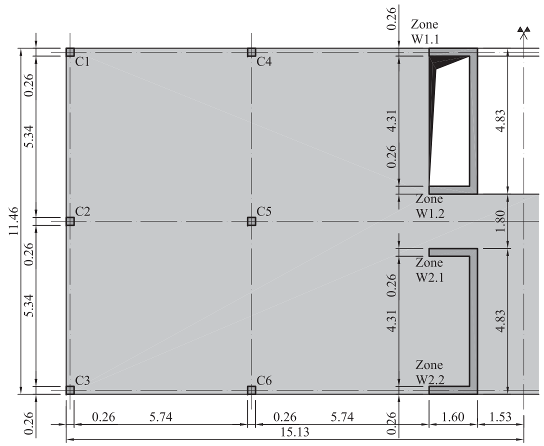
1 Basic data

1.1 Geometry (dimensions in [m])

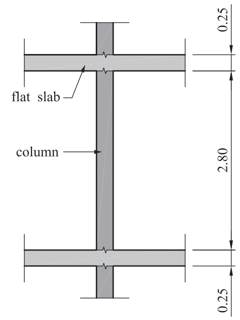


Spans: L_x = 6.00 m and L_y = 5.60 m

Plan view



Section trough slab and column



Slab thickness h: 25 cm

Cover concrete c: 3 cm

1.2 Material

The material properties can be found in chapter 5 of model code 2010.

Concrete C30		Steel B500S (flexural and transverse reinforcement)	
f_{ck}	30 MPa	f_{yd}	435 MPa
γ_c	1.5	E_s	200 GPa
d_g	32 mm	Ductility class	B

1.3 Loads

Self-weight of concrete slab: 6.25 kN/m²

Superimposed dead load: 2 kN/m²

Live load: 3 kN/m²

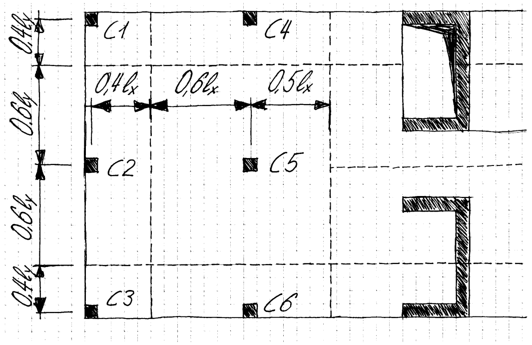
$$g_d + q_d = 1.35(6.25 + 2) + 1.5 \cdot 3 = 15.6\,\text{kN/m}^2$$

2 Level I of approximation (preliminary design)

The goal of the preliminary design is to check if the dimensions of the structure are reasonable with respect to the punching shear strength and if punching shear reinforcement is needed.

Reaction forces

The reaction forces in the columns are estimated by using contributive areas.



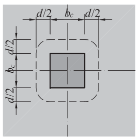
Inner (C5): $V_d \approx 692$ kN
Corner (C1, C3): $V_d \approx 93$ kN
Edge (C2): $V_d \approx 265$ kN

(C4 and C6 are not governing $V_d \approx 244$ kN)

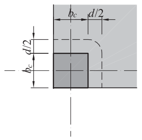
Control perimeter

The effective depth d_v is assumed to be 200 mm.

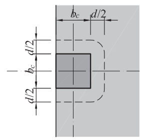
Eccentricity coefficient (k_e) are adopted from the commentary of §7.3.5.2



Inner column
(C5)
k_e=0.9



Corner Column
(C1, C3)
k_e=0.65



Edge Column
(C2, C4, C6)
k_e=0.7

Inner: $$b_0 = k_e \cdot (4 \cdot b_c + d_v \cdot \pi) = 0.90 \cdot (4 \cdot 260 + 200 \cdot \pi) = 1501\,\text{mm}$$

Corner: $$b_0 = k_e \cdot \left(2 \cdot b_c + \frac{d_v \cdot \pi}{4}\right) = 0.65 \cdot \left(2 \cdot 260 + \frac{200 \cdot \pi}{4}\right) = 440\,\text{mm}$$

Edge: $$b_0 = k_e \cdot \left(3 \cdot b_c + \frac{d_v \cdot \pi}{2}\right) = 0.70 \cdot \left(3 \cdot 260 + \frac{200 \cdot \pi}{2}\right) = 766\,\text{mm}$$

Rotations

According to the commentary, the distance to the point where the radial moment is zero r_s can be estimated based on the spans.

By using the Level I approach, one can estimate the rotations.

$$r_{s,x} = 0.22 L_x = 0.22 \cdot 6.0 = 1.32\,\text{m} \qquad r_{s,y} = 0.22 L_y = 0.22 \cdot 5.6 = 1.23\,\text{m}$$

$$\psi_x = 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{1.32}{0.200} \frac{435}{200000} = 0.0215 \Rightarrow \text{governing}$$

$$\psi_y = 1.5 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{1.23}{0.200} \frac{435}{200000} = 0.0200$$

The maximum aggregate size of 32 mm leads to a factor k_{dg} of

$$k_{dg} = \frac{48}{16 + d_g} = \frac{48}{16 + 32} = 1.0$$

$$k_\psi = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.0215 \cdot 200 \cdot 1.0} = 0.25 \le 0.6$$

To check if shear reinforcement and which system can be used, one can calculate the minimal needed value of factor k_{sys}.

$$V_{Rd,\max} = k_{sys} k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v \geq V_d \rightarrow k_{sys} \geq \frac{V_d}{k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 d_v} = \frac{V_d}{V_{Rd,c}}$$

k_{sys} depends on the performance of the used shear reinforcement system. The model code proposes a value of $k_{sys} = 2.0$ for system compliant with model code detailing rules (§7.13.5.3). Higher values (up to $k_{sys} = 2.8$) can be used if more restrictive detailing rules are adopted and if the placing of the transverse reinforcement is checked at the construction site.

Shear strength without shear reinforcement

Inner: $V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0\ d_v = 0.25 \frac{\sqrt{30}}{1.5} 1501 \cdot 200 \cdot 10^{-3} = 274 \text{ kN} < V_d = 692 \text{ kN}$

Corner: $V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0\ d_v = 0.25 \frac{\sqrt{30}}{1.5} 440 \cdot 200 \cdot 10^{-3} = 80 \text{ kN} < V_d = 93 \text{ kN}$

Edge: $V_{Rd,c} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0\ d_v = 0.25 \frac{\sqrt{30}}{1.5} 766 \cdot 200 \cdot 10^{-3} = 140 \text{ kN} < V_d = 265 \text{ kN}$

The thickness of the slab has to be increased or the slab has to be shear reinforced.

Shear reinforcement

Inner: $k_{sys} \geq \frac{V_d}{V_{Rd,c}} = \frac{692}{274} = 2.5$

Corner: $k_{sys} \geq \frac{V_d}{V_{Rd,c}} = \frac{93}{80} = 1.2$

Edge: $k_{sys} \geq \frac{V_d}{V_{Rd,c}} = \frac{265}{140} = 1.9$

Conclusions

Inner column: Shear reinforcement is necessary and sufficient (accounting for the values of k_{sys}) to ensure punching shear strength

Corner columns: Shear reinforcement might probably not be necessary. This has to be confirmed by a higher level of approximation.

Edge columns: Shear reinforcement might probably be necessary.

The thickness of the slab is sufficient if shear reinforcement is used.

The moments and the reaction forces have been calculated with a finite element software. For the analysis, a linear-elastic model has been used.

The moment M_d is the vector addition of the moments in x- and y-direction.

$$M_d = \sqrt{M_{d,x}^2 + M_{d,y}^2}$$

For a level II approximation, one has to know the flexural reinforcement. It was designed on the basis of the previous finite element analysis.

The flexural strength can be calculated according to the Model Code. In this example, however, the flexural strength has been calculated assuming a rigid-plastic behavior of concrete and steel:

$$m_{Rd} = \rho \cdot d^2 \cdot f_{yd} \left(1 - \frac{\rho \cdot f_{yd}}{2 f_{cd}}\right)$$

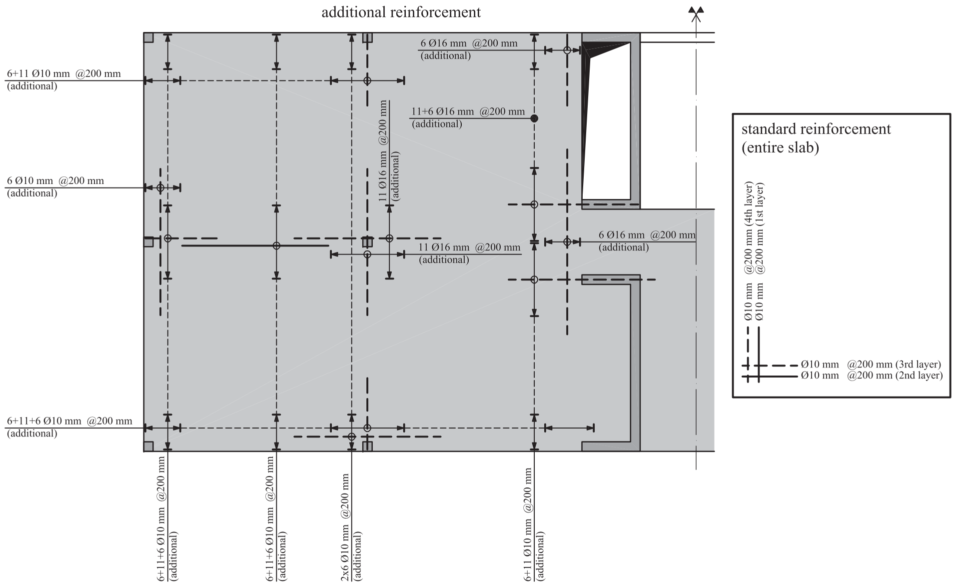
3 Level II of approximation (typical design)

3.4 Structural analysis and flexural design

Summary of the column reactions

Column	R_d [kN]	$M_{d,x,}$ [kNm]	$M_{d,y}$ [kNm]	M_d [kNm]
C1	111	25	22	33
C2	266	42	0	42
C3	112	25	22	33
C4	252	3	36	36
C5	664	8	1	8
C6	246	5	34	34

Reinforcement sketch



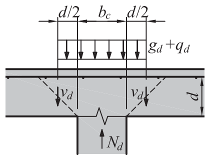
Flexural strength

ø10 @200 mm	m_{Rd} = 35 kNm/m	d = 210 mm
ø10 @100 mm	m_{Rd} = 69 kNm/m	d = 210 mm
ø10 @200 mm / ø16 @200 mm	m_{Rd} = 115 kNm/m	d = 204 mm

3.5 Shear design inner column C5

Design shear force

The design shear force V_d is equal to the column reaction force N_d minus the applied load within the control perimeter $(g_d + q_d)\cdot A_c$.



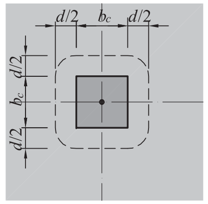
$$A_c = b_c^2 + 2\cdot b_c \cdot d_v + \frac{d_v^2}{4}\pi = 0.26^2 + 2\cdot 0.26\cdot 0.204 + \frac{0.204^2}{4}\pi = 0.21\,m^2$$

$$V_d = N_d - \left(g_d + q_d\right)\cdot A_c = 664 - 15.6\cdot 0.21 = 661 \text{ kN}$$

Control perimeter

In case of inner columns, the centroid of the column corresponds to the centroid of the control perimeter. Therefore,

$\Delta e = 0$



$$e_u = \left|\frac{M_d}{V_d} - \Delta e\right| = \left|\frac{8\cdot 10^6}{661\cdot 10^3} - 0\right| = 12\,\text{mm} \qquad b_u = \sqrt{\frac{4}{\pi}A_c} = \sqrt{\frac{4}{\pi}206\cdot 10^3} = 513\,\text{mm}$$

$$k_e = \frac{1}{1 + e_u/b_u} = \frac{1}{1 + 12/513} = 0.98$$

$$b_0 = k_e \cdot b_1 = k_e \cdot \left(4\cdot b_c + d_v \cdot \pi\right) = 0.98\cdot\left(4\cdot 260 + 204\cdot\pi\right) = 1642\,\text{mm}$$

Rotations

The distances $r_{s,x}$ and $r_{s,y}$ are the same as for the Level I approximation.

$$r_{s,x} = 1.32\,\text{m} \qquad r_{s,y} = 1.23\,\text{m}$$

$$b_s = 1.5\cdot\sqrt{r_{s,x}\cdot r_{s,y}} = 1.5\cdot\sqrt{1.32\cdot 1.23} = 1.91\,\text{m}$$

$$m_{sd,x} = \frac{V_d}{8} + \left|\frac{M_{d,x} - V_d \cdot \Delta e_x}{2\cdot b_s}\right| = \frac{661}{8} + \left|\frac{8}{2\cdot 1.91}\right| = 85\,\text{kNm/m}$$

$$m_{sd,y} = \frac{V_d}{8} + \left|\frac{M_{d,y} - V_d \cdot \Delta e_y}{2\cdot b_s}\right| = \frac{661}{8} + \left|\frac{1}{2\cdot 1.91}\right| = 83\,\text{kNm/m}$$

$$\psi_x = 1.5\cdot\frac{r_{s,x}}{d}\frac{f_{yd}}{E_s}\cdot\left(\frac{m_{sd}}{m_{Rd,x}}\right)^{1.5} = 1.5\cdot\frac{1.32}{0.204}\frac{435}{200000}\cdot\left(\frac{85}{115}\right)^{1.5} = 0.0133 \Rightarrow \text{governing}$$

$$\psi_y = 1.5\cdot\frac{r_{s,y}}{d}\frac{f_{yd}}{E_s}\cdot\left(\frac{m_{sd}}{m_{Rd,y}}\right)^{1.5} = 1.5\cdot\frac{1.23}{0.204}\frac{435}{200000}\cdot\left(\frac{83}{115}\right)^{1.5} = 0.0121$$

k_{dg} is calculated at Level I.

$$k_\psi = \frac{1}{1.5 + 0.6\cdot\psi\cdot d\cdot k_{dg}} = \frac{1}{1.5 + 0.6\cdot 0.0133\cdot 204\cdot 1.0} = 0.32 < 0.6$$

Punching strength without shear reinforcement

$$V_{Rd,c} = k_{\psi}\frac{\sqrt{f_{ck}}}{\gamma_c}b_0\ d_v = 0.32\frac{\sqrt{30}}{1.5}1642\cdot 204\cdot 10^{-3} = 390\text{ kN} < V_d = 661\text{kN}$$

The punching shear strength of the concrete is not sufficient. Consequently, shear reinforcement is necessary.

Punching strength with shear reinforcement

Firstly, one has to check if the design shear force V_d is smaller than the maximum punching strength $V_{Rd,max}$. This is done assuming k_{sys}=2.

$$V_{Rd,\max} = k_{sys}V_{Rd,c} = 2\cdot 390 = 781\text{kN} \le \frac{\sqrt{f_{ck}}}{\gamma_c}b_0 d_v = 1223\text{kN}$$

The design shear force V_d is below the maximum punching strength $V_{Rd,max}$. Therefore, the slab can be reinforced with shear reinforcement complying with detailing rules defined in subclause §7.13.5.3.

$$V_{Rd,\max} = 781\text{kN} \ge V_d = 661\text{kN}$$

$$\sigma_{swd} = \frac{E_s\psi}{6}\cdot\left(1+\frac{f_{bd}}{f_{ywd}}\cdot\frac{d}{\phi_w}\right) = \frac{200000\cdot 0.0133}{6}\cdot\left(1+\frac{3}{435}\cdot\frac{204}{8}\right)$$

The bond strength is taken as f_{bd} = 3 MPa (according to MC 2010 for corrugated bars).

$$\sigma_{swd} = 521\text{MPa} > f_{ywd} = 435\text{MPa}$$

$$A_{sw} = \frac{V_d - V_{Rd,c}}{k_e\sigma_{swd}\sin\alpha} = \frac{(661-390)\cdot 10^3}{0.98\cdot 435\cdot\sin(90°)} = 636\text{mm}^2$$

$$A_{sw,\min} = \frac{0.5\cdot V_d}{k_e\cdot f_{ywd}\cdot\sin(\alpha)} = \frac{0.5\cdot 661}{0.98\cdot 435\cdot\sin(90°)} = 774\text{mm}^2 \Rightarrow \text{governing}$$

To avoid a failure outside the shear reinforced area, the outer perimeter need to have a minimal length. The design shear force can be reduced to account for the loads applied inside the outer perimeter. This effect is neglected as a safe estimate.

$$b_0 = \frac{V_d}{k_{\psi}\frac{\sqrt{f_{ck}}}{\gamma_c}d_{v,out}} = \frac{661\cdot 10^3}{0.32\frac{\sqrt{30}}{1.5}174} = 3258\text{mm}$$

In this example, the calculating value of the effective depth d_v is equal to the effective depth d minus the concrete cover c on the bottom surface of the slab.

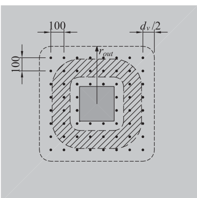
$$d_{v,out} = d - c = 204 - 30 = 174\,mm$$

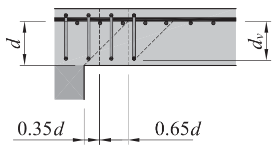
Assuming a circular control perimeter for the estimation of the eccentricity, factor k_e can be estimated as detailed in the right hand side column.

$$r_{out} = \frac{b_0}{2\pi} = \frac{3258}{2\pi} = 519\text{mm} \qquad k_e = \frac{1}{1+e_u/2r_{out}} = \frac{1}{1+12/(2\cdot 519)} = 0.99$$

$$b_{out} = \frac{b_0}{k_e} = \frac{3258}{0.99} = 3296\text{mm}$$

Possible shear reinforcement layout:





$$\rho_{sw} = \varnothing 8@100@100 = 0.50\%$$

$$A_{sw} = \rho_w\left[4\cdot b_c\cdot d_v + d_v^2\cdot\pi - 4\cdot b_c\cdot 0.35d_v - (0.35d_v)^2\cdot\pi\right]$$

$$A_{sw} = 0.005\left[4\cdot 260\cdot 204 + 204^2\cdot\pi - 4\cdot 260\cdot 0.35\cdot 204 - (0.35\cdot 204)^2\cdot\pi\right]$$

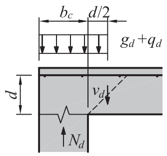
$$A_{sw} = 1263\text{mm}^2 > 774\text{mm}^2$$

$$b_{out} = 4\cdot 700 + 174\cdot\pi = 3347\text{ mm} > 3296\text{mm}$$

3.6 Shear design corner column C1 and C3

Design shear force

The design shear force V_d is equal to the column reaction force N_d minus the applied load within the control perimeter $(g_d + q_d)\cdot A_c$.

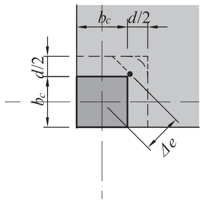


$$A_c = b_c^2 + 2\cdot b_c \cdot \frac{d_v}{2} + \frac{d_v^2}{16}\pi = 0.26^2 + 0.26\cdot 0.210 + \frac{0.210^2}{16}\pi = 0.13\,m^2$$

$$V_d = N_d - \left(g_d + q_d\right)\cdot A_c = 112 - 15.6\cdot 0.13 = 110 \text{ kN}$$

Control perimeter

$$\Delta e_x = \Delta e_y = \frac{3}{4}\left(b_c + \frac{d_v}{2}\right) - \frac{b_c}{2} = \frac{1}{4}\left(b_c + \frac{3}{2}d_v\right)$$



$$\Delta e_x = \Delta e_y = \frac{1}{4}\left(b_c + \frac{3}{2}d_v\right) = \frac{1}{4}\left(260 + \frac{3}{2}210\right) = 144 \text{ mm}$$

$$\Delta e = \Delta e_x \cdot \sqrt{2} = 144\cdot\sqrt{2} = 203\,\text{mm}$$

$$e_u = \left|\frac{M_d}{V_d} - \Delta e\right| = \left|\frac{33\cdot 10^6}{110\cdot 10^3} - 203\right| = 97\,\text{mm} \qquad b_u = \sqrt{\frac{4}{\pi}A_c} = \sqrt{\frac{4}{\pi}131\cdot 10^3} = 408\text{mm}$$

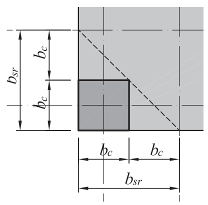
$$k_e = \frac{1}{1 + e_u/b_u} = \frac{1}{1 + 97/408} = 0.81$$

$$b_0 = k_e \cdot b_1 = k_e \cdot \left(2\cdot b_c + \frac{d_v \cdot \pi}{4}\right) = 0.81\cdot\left(2\cdot 260 + 210\cdot\frac{\pi}{4}\right) = 554\,\text{mm}$$

Rotations

The distances $r_{s,x}$ and $r_{s,y}$ are the same as for the Level I approximation.

In case of corner columns, the width of the support strip may be limited by the distance b_{sr}.



$$r_{s,x} = 1.32\,\text{m} \qquad r_{s,y} = 1.23\,\text{m}$$

$$b_s = 1.5\cdot\sqrt{r_{s,x}\cdot r_{s,y}} = 1.5\cdot\sqrt{1.32\cdot 1.23} = 1.91\,\text{m}$$

$$b_{sr} = 2\cdot b_c = 2\cdot 0.26 = 0.52\,\text{m} \Rightarrow \text{governing}$$

$$m_{sd,x} = \frac{V_d}{8} + \left|\frac{M_{d,x} - V_d\cdot\Delta e_x}{b_s}\right| = \frac{110}{8} + \left|\frac{25 - 110\cdot 0.144}{0.52}\right| = 31\,\text{kN} < \frac{V_d}{2} = \frac{110}{2} = 55\,\text{kN}$$

$$m_{sd,y} = \frac{V_d}{8} + \left|\frac{M_{d,y} - V_d\cdot\Delta e_y}{b_s}\right| = \frac{110}{8} + \left|\frac{22 - 110\cdot 0.144}{0.52}\right| = 25\,\text{kN} < \frac{V_d}{2} = \frac{110}{2} = 55\,\text{kN}$$

$$\psi_x = 1.5 \cdot \frac{r_{s,x}}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd,x}}{m_{Rd,x}}\right)^{1.5} = 1.5 \cdot \frac{1.32}{0.21}\frac{435}{200000} \cdot \left(\frac{55}{69}\right)^{1.5} = 0.0146 \Rightarrow \text{governing}$$

$$\psi_y = 1.5 \cdot \frac{r_{s,y}}{d}\frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd,y}}{m_{Rd,y}}\right)^{1.5} = 1.5 \cdot \frac{1.23}{0.21}\frac{435}{200000} \cdot \left(\frac{55}{69}\right)^{1.5} = 0.0136$$

k_{dg} is calculated at Level I.

$$k_\psi = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.0146 \cdot 210 \cdot 1} = 0.30 < 0.6$$

Punching strength without shear reinforcement

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary

$$V_{Rd} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0\, d_v = 0.30 \frac{\sqrt{30}}{1.5} 554 \cdot 210 \cdot 10^{-3} = 127\,\text{kN} > 110\,\text{kN}$$

Integrity reinforcement

Since no shear reinforcement has been used and $m_{sd} < m_{Rd}$, integrity reinforcement needs to be provided.

For the design of the integrity reinforcement, the accidental load case can be used. Thus, the design load can be reduced.

$$(g_d + q_d)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1\text{kN/m}^2$$

$$V_{d,acc} = \frac{(g_d + q_d)_{acc}}{g_d + q_d} \cdot V_d = \frac{10.1}{15.6} \cdot 110 = 71\text{kN}$$

The material properties can be found in chapter 5 of model code 2010.

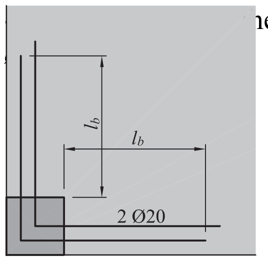
Ductility class B : $(f_t/f_y)_k = 1.08$ and $\varepsilon_{uk} = 0.05$

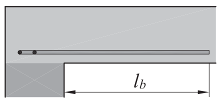
It is assumed that only straight bars will be used, thus $\alpha = 0°$.

With respect to integrity reinforcement, two restrictions should be fulfilled:

-the integrity reinforcement should at least be composed of four bars

he integrity bars $ø_{int}$ has to be chosen such that





$$A_s = \frac{V_{d,acc}}{f_{yd} \cdot (f_t / f_y)_k \cdot \sqrt{1 - \left(\frac{\cos\alpha}{1 + 1.25\varepsilon_{uk}^{1.5}}\right)^2}} = \frac{71 \cdot 10^3}{435 \cdot 1.08 \cdot \sqrt{1 - \left(\frac{\cos(0)}{1 + 1.25 \cdot 0.05^{1.5}}\right)^2}} = 916\,\text{mm}^2$$

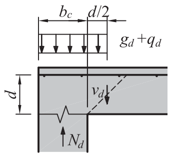
⇨ 2x2 ø20 $A_s = 1257$ mm^2 (2 in each direction)

$$0.12 d_{res} = 0.12\left(h - 2 \cdot c - \phi_{top} - \phi_{bottom}\right) = 0.12\left(250 - 2 \cdot 30 - 10 - 20\right) = 19\,\text{mm} \approx 20\,\text{mm}$$

3.7 Shear design edge column C2

Design shear force

The design shear force V_d is equal to the column reaction force N_d minus the applied load within the control perimeter $(g_d + q_d)\cdot A_c$.



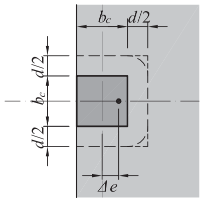
$$A_c = b_c^2 + 3\cdot b_c \cdot \frac{d_v}{2} + \frac{d_v^{\,2}}{8}\pi = 0.26^2 + \frac{3}{2}0.26\cdot 0.21 + \frac{0.21^2}{8}\pi = 0.17\,\text{m}^2$$

$$V_d = N_d - \left(g_d + q_d\right)\cdot A_c = 266 - 15.6\cdot 0.17 = 263\ \text{kN}$$

Control perimeter

$$\Delta e_x = \frac{\left(b_c + d_v\right)\cdot\left(b_c + {d_v}/{2}\right) + 2\left(b_c + {d_v}/{2}\right)\cdot\frac{1}{2}\left(b_c + {d_v}/{2}\right)}{\left(b_c + d_v\right) + 2\left(b_c + {d_v}/{2}\right)} - \frac{b_c}{2}$$

$$\Delta e_x = \frac{1}{4}\cdot\frac{2b_c^2 + 6b_c d_v + 3d_v^2}{3b_c + 2d_v}$$



$$\Delta e_x = \frac{1}{4}\cdot\frac{2b_c^2 + 6b_c d_v + 3d_v^2}{3b_c + 2d_v} = \frac{1}{4}\cdot\frac{2\cdot 260^2 + 6\cdot 260\cdot 210 + 3\cdot 210^2}{3\cdot 260 + 2\cdot 210} = 124\,\text{mm}$$

$$\Delta e_y = 0\,\text{mm} \qquad \Delta e = \Delta e_x = 124\,\text{mm}$$

$$e_u = \left|\frac{M_d}{V_d} - \Delta e\right| = \left|\frac{42\cdot 10^6}{263\cdot 10^3} - 124\right| = 35\,\text{mm} \qquad b_u = \sqrt{\frac{4}{\pi}A_c} = \sqrt{\frac{4}{\pi}167\cdot 10^3} = 461\,\text{mm}$$

$$k_e = \frac{1}{1 + e_u/b_u} = \frac{1}{1 + 35/461} = 0.93$$

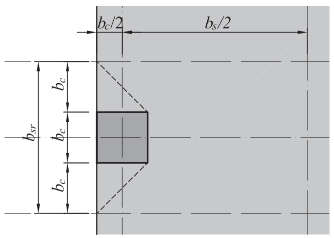
$$b_0 = k_e \cdot b_1 = k_e \cdot\left(3\cdot b_c + \frac{d_v\cdot\pi}{2}\right) = 0.93\cdot\left(3\cdot 260 + 210\cdot\frac{\pi}{2}\right) = 1031\,\text{mm}$$

Rotations

The distances $r_{s,x}$ and $r_{s,y}$ are the same as for the Level I approximation.

$$r_{s,x} = 1.32\,\text{m} \qquad r_{s,y} = 1.23\,\text{m}$$

In case of edge columns, the width of the support strip may be limited by the distance b_{sr}.



$$b_s = 1.5\cdot\sqrt{r_{s,x}\cdot r_{s,y}} = 1.5\cdot\sqrt{1.32\cdot 1.23} = 1.91\,\text{m}$$

$$b_{sr,x} = 3\cdot b_c = 3\cdot 0.26 = 0.78\,\text{m} \ \Rightarrow \text{governing}$$

$$b_{sr,y} = \frac{b_c}{2} + \frac{b_s}{2} = \frac{0.260}{2} + \frac{1.91}{2} = 1.09\,\text{m} \ \Rightarrow \text{governing}$$

$$m_{sd,x} = \frac{V_d}{8} + \left|\frac{M_{d,x} - V_d\cdot\Delta e_x}{b_s}\right| = \frac{263}{8} + \left|\frac{42 - 263\cdot 0.124}{0.78}\right| = 45\,\text{kN}$$

$$m_{sd,y} = \frac{V_d}{8} + \left|\frac{M_{d,y} - V_d\cdot\Delta e_y}{2\cdot b_s}\right| = \frac{263}{8} + \left|\frac{0}{2\cdot 1.09}\right| = 33\,\text{kN} < \frac{V_d}{4} = \frac{263}{4} = 66\,\text{kN}$$

$$\psi_x = 1.5 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd,x}}{m_{Rd,x}} \right)^{1.5} = 1.5 \cdot \frac{1.32}{0.21} \frac{435}{200000} \cdot \left(\frac{45}{69} \right)^{1.5} = 0.011$$

$$\psi_y = 1.5 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd,y}}{m_{Rd,y}} \right)^{1.5} = 1.5 \cdot \frac{1.23}{0.21} \frac{435}{200000} \cdot \left(\frac{66}{69} \right)^{1.5} = 0.018 \Rightarrow \text{governing}$$

k_{dg} is calculated at Level I.

$$k_\psi = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.018 \cdot 210 \cdot 1.0} = 0.27 < 0.6$$

Punching strength without shear reinforcement

$$V_{Rd} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 \, d_v = 0.27 \frac{\sqrt{30}}{1.5} 1031 \cdot 210 \cdot 10^{-3} = 211\,\text{kN} < 263\,\text{kN}$$

The punching shear strength of the concrete is not sufficient. Since the strength seems to be rather close to the design load, a level III approximation will be performed.

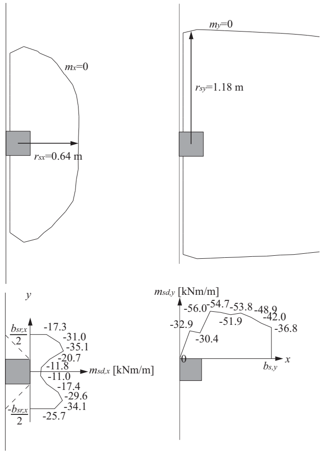
4 Level III of approximation (detailed design or assessment of existing structure)

The Level III calculations are based on the results of the linear-elastic finite element analysis.

From the results of the flexural analysis, one can obtain the distance between the center of the column and the point, at which the bending moments are zero.

The average moment in the support strip can be obtained by the integration of the moments at the strip section.

Since the flexural moments $m_{d,x}$ and $m_{d,y}$ at the support regions are negative, the absolute value of the twisting moment $m_{d,x}$ need to be subtracted so that the absolute value of $m_{sd,x}$ and $m_{sd,y}$ will be maximized.



$$m_{sd,x} = m_{d,x} - \left|m_{d,xy}\right|$$

$$m_{sd,y} = m_{d,y} - \left|m_{d,xy}\right|$$

$r_{s,x} = 0.64\,\text{m}$ $\qquad$ $r_{s,y} = 1.18\,\text{m}$

$$b_s = 1.5 \cdot \sqrt{r_{s,x} \cdot r_{s,y}} = 1.5 \cdot \sqrt{0.64 \cdot 1.18} = 1.30\,\text{m}$$

$$b_{sr,x} = 3 \cdot b_c = 3 \cdot 0.26 = 0.78\,\text{m}$$

$$b_{sr,y} = \frac{b_c}{2} + \frac{b_s}{2} = \frac{0.26}{2} + \frac{1.30}{2} = 0.78\,\text{m}$$

$m_{sd,x} = 24\,\text{kNm/m}$ $\qquad$ $m_{sd,y} = 43\,\text{kNm/m}$ $\qquad$ (average value on support strip)

$r_{s,x} = 0.64\text{m} > \frac{2}{3} b_{sr,x} = 0.52\,\text{m}$ $\qquad$ $r_{s,y} = 1.18\,\text{m} > \frac{2}{3} b_{sr,y} = 0.52\,\text{m}$

$$\psi_x = 1.2 \cdot \frac{r_{s,x}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd}}{m_{Rd,x}}\right)^{1.5} = 1.2 \cdot \frac{0.64}{0.21} \frac{435}{200000} \cdot \left(\frac{24}{69}\right)^{1.5} = 0.0016$$

$$\psi_y = 1.2 \cdot \frac{r_{s,y}}{d} \frac{f_{yd}}{E_s} \cdot \left(\frac{m_{sd}}{m_{Rd,y}}\right)^{1.5} = 1.2 \cdot \frac{1.18}{0.21} \frac{435}{200000} \cdot \left(\frac{43}{69}\right)^{1.5} = 0.0073 \;\Rightarrow\; \text{governing}$$

$$k_\psi = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.0073 \cdot 210 \cdot 1.0} = 0.41 < 0.6$$

$$V_{Rd} = k_\psi \frac{\sqrt{f_{ck}}}{\gamma_c} b_0 \; d = 0.41 \frac{\sqrt{30}}{1.5} 1047 \cdot 210 \cdot 10^{-3} = 332\,\text{kN} > 263\,\text{kN}$$

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary

Integrity reinforcement

Since no shear reinforcement has been used and $m_{sd} < m_{Rd}$, integrity reinforcement needs to be provided to prevent a progressive collapse of the structure.

For the design of the integrity reinforcement, the accidental load case can be used. Thus the design load can be reduced.

$$\left(g_d + q_d\right)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1\,\text{kN/m}^2$$

$$V_{d,acc} = \frac{\left(g_d + q_d\right)_{acc}}{g_d + q_d} \cdot V_d = \frac{10.1}{15.6} \cdot 263 = 171\,\text{kN}$$

$$A_s = \frac{V_{d,acc}}{f_{yd} \cdot \left(f_t / f_y\right)_k \cdot \sqrt{1 - \left(\frac{\cos\alpha}{1 + 1.25\varepsilon_{uk}^{1.5}}\right)^2}} = \frac{171 \cdot 10^3}{435 \cdot 1.08 \cdot \sqrt{1 - \left(\frac{\cos(0)}{1 + 1.25 \cdot 0.05^{1.5}}\right)^2}} = 2194\,\text{mm}^2$$

⇨ 2x3ø20 + 3ø16 A_s = 2488 mm^2 (3 in each direction)

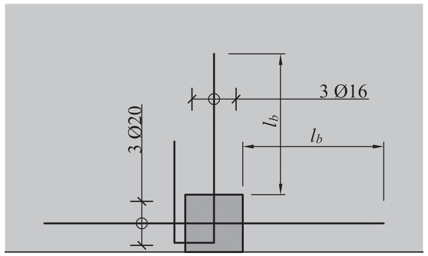
$$0.12 d_{res} = 0.12\left(h - 2 \cdot c - \phi_{top} - \phi_{bottom}\right) = 0.12\left(250 - 2 \cdot 30 - 10 - 20\right) = 19\,\text{mm} \approx 20\,\text{mm}$$

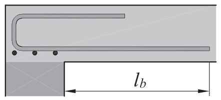
The material properties can be found in chapter 5 of model code 2010.

Ductility class B : $(f_t/f_y)_k = 1.08$ and $\varepsilon_{uk} = 0.05$

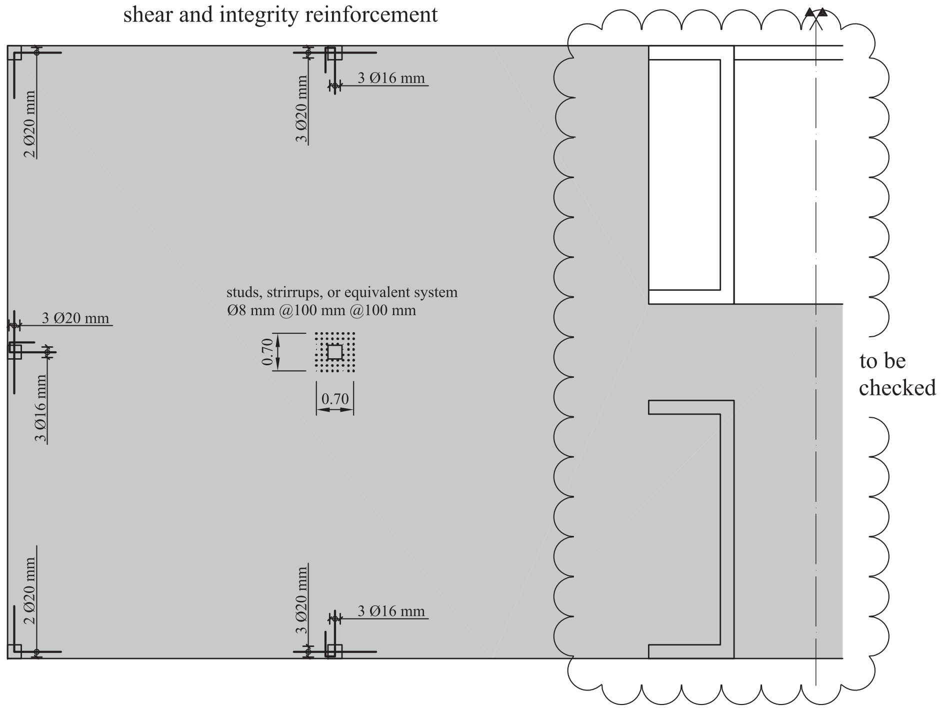
It is assumed that only straight bars will be used, thus $\alpha = 0°$.

With respect to integrity reinforcement, two restrictions should be fulfilled:
-the integrity reinforcement should at least be composed of four bars
-the diameter of the integrity bars $ø_{int}$ has to be chosen such that
$ø_{int} \leq 0.12 \cdot d_{res}$





Corners of walls should be checked following the same methodology.



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