Punching of flat slabs: Design example

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1 Basic data

1.1 Geometry (dimensions in [m])

Plan view

Section through slab and column

Spans: \( L_x = 6.00 \text{ m} \) and \( L_y = 5.60 \text{ m} \)

The material properties can be found in chapter 5 of model code 2010.

1.2 Material

Concrete C30

Steel B500S (flexural and transverse reinforcement)

\[
\begin{align*}
  f_{ck} & = 30 \text{ MPa} \\
  f_{yd} & = 435 \text{ MPa} \\
  \gamma_c & = 1.5 \\
  E_s & = 200 \text{ GPa} \\
  d_g & = 32 \text{ mm} \\
\end{align*}
\]

Ductility class B

1.3 Loads

Self-weight of concrete slab: \( 6.25 \text{ kN/m}^2 \)
Superimposed dead load: \( 2 \text{ kN/m}^2 \)
Live load: \( 3 \text{ kN/m}^2 \)

\[
g_d + q_d = 1.35(6.25 + 2) + 1.5 \cdot 3 = 15.6 \text{ kN/m}^2
\]
The goal of the preliminary design is to check if the dimensions of the structure are reasonable with respect to the punching shear strength and if punching shear reinforcement is needed.

The reaction forces in the columns are estimated by using contributive areas.

The effective depth $d_e$ is assumed to be 200 mm.

Eccentricity coefficient ($k_e$) are adopted from the commentary of §7.3.5.2

According to the commentary, the distance to the point where the radial moment is zero $r_s$ can be estimated based on the spans.

By using the Level I approach, one can estimate the rotations.

The maximum aggregate size of 32 mm leads to a factor $k_{dg}$ of

$$k_{dg} = \frac{48}{16 + d_s} = \frac{48}{16 + 32} = 1.0$$

2 Level I of approximation (preliminary design)

Reaction forces

Inner (C5): $V_d \approx 692$ kN
Corner (C1, C3): $V_d \approx 93$ kN
Edge (C2): $V_d \approx 265$ kN

(C4 and C6 are not governing $V_d \approx 244$ kN)

Control perimeter

Inner: $b_o = k_e \cdot (4 \cdot b_s + d_s \cdot \pi) = 0.90 \cdot (4 \cdot 260 + 200 \cdot \pi) = 1501$ mm
Corner: $b_o = k_e \cdot \left(2 \cdot b_s + \frac{d_s \cdot \pi}{4}\right) = 0.65 \cdot \left(2 \cdot 260 + \frac{200 \cdot \pi}{4}\right) = 440$ mm
Edge: $b_o = k_e \cdot \left(3 \cdot b_s + \frac{d_s \cdot \pi}{2}\right) = 0.70 \cdot \left(3 \cdot 260 + \frac{200 \cdot \pi}{2}\right) = 766$ mm

Rotations

$r_{sx} = 0.22L_s = 0.22 \cdot 6.0 = 1.32$ m $r_{sy} = 0.22L_s = 0.22 \cdot 5.6 = 1.23$ m

$\psi_x = 1.5 \cdot \frac{r_{sx}}{d_s} \cdot \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{1.32}{200} \cdot \frac{435}{200000} = 0.0215$ governing

$\psi_y = 1.5 \cdot \frac{r_{sy}}{d_s} \cdot \frac{f_{yd}}{E_s} = 1.5 \cdot \frac{1.23}{200} \cdot \frac{435}{200000} = 0.0200$

$k_{df} = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.2015 \cdot 200 \cdot 1.0} = 0.25 \leq 0.6$
Shear strength without shear reinforcement

Inner: \( V_{sd, i} = k_p \sqrt{f_{ct}} b_i d_i = 0.25 \times \frac{30}{1.5} = 274 \text{kN} < V_d = 692 \text{kN} \)

Corner: \( V_{sd, c} = k_p \sqrt{f_{ct}} b_c d_c = 0.25 \times \frac{30}{1.5} = 80 \text{kN} < V_d = 93 \text{kN} \)

Edge: \( V_{sd, e} = k_p \sqrt{f_{ct}} b_e d_e = 0.25 \times \frac{30}{1.5} = 140 \text{kN} < V_d = 265 \text{kN} \)

The thickness of the slab has to be increased or the slab has to be shear reinforced.

Shear reinforcement

Inner:
\[
k_{sys} \geq \frac{V_c}{V_{sd, i}} \geq \frac{692}{274} = 2.5
\]

Corner:
\[
k_{sys} \geq \frac{V_c}{V_{sd, c}} \geq \frac{93}{80} = 1.2
\]

Edge:
\[
k_{sys} \geq \frac{V_c}{V_{sd, e}} \geq \frac{265}{140} = 1.9
\]

Conclusions

Inner column: Shear reinforcement is necessary and sufficient (accounting for the values of \( k_{sys} \)) to ensure punching shear strength. 

Corner columns: Shear reinforcement might probably not be necessary. This has to be confirmed by a higher level of approximation. 

Edge columns: Shear reinforcement might probably be necessary.

The thickness of the slab is sufficient if shear reinforcement is used.
The moments and the reaction forces have been calculated with a finite element software. For the analysis, a linear-elastic model has been used. The moment $M_d$ is the vector addition of the moments in x- and y-direction.

$$M_d = \sqrt{M_{d,x}^2 + M_{d,y}^2}$$

For a level II approximation, one has to know the flexural reinforcement. It was designed on the basis of the previous finite element analysis.

The flexural strength can be calculated according to the Model Code. In this example, however, the flexural strength has been calculated assuming a rigid-plastic behavior of concrete and steel:

$$m_{rd} = \rho \cdot d^2 \cdot f_{yd} \left(1 - \frac{\rho \cdot f_{yd}}{2f_{yd}}\right)$$

### 3 Level II of approximation (typical design)

#### 3.4 Structural analysis and flexural design

<table>
<thead>
<tr>
<th>Column</th>
<th>$R_d$ [kN]</th>
<th>$M_{d,x}$ [kNm]</th>
<th>$M_{d,y}$ [kNm]</th>
<th>$M_d$ [kNm]</th>
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<tbody>
<tr>
<td>C1</td>
<td>111</td>
<td>25</td>
<td>22</td>
<td>33</td>
</tr>
<tr>
<td>C2</td>
<td>266</td>
<td>42</td>
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<td>C3</td>
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<td>C5</td>
<td>664</td>
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<tr>
<td>C6</td>
<td>246</td>
<td>5</td>
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</tbody>
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The moments and reaction forces have been calculated with a finite element software. For the analysis, a linear-elastic model has been used. The moment $M_d$ is the vector addition of the moments in x- and y-direction. $M_d = \sqrt{M_{d,x}^2 + M_{d,y}^2}$

For a level II approximation, one has to know the flexural reinforcement. It was designed on the basis of the previous finite element analysis.

The flexural strength can be calculated according to the Model Code. In this example, however, the flexural strength has been calculated assuming a rigid-plastic behavior of concrete and steel:

$m_{rd} = \rho \cdot d^2 \cdot f_{yd} \left(1 - \frac{\rho \cdot f_{yd}}{2f_{yd}}\right)$

### Flexural strength

- $\Phi10 \ @200$ mm $m_{rd} = 35$ kNm/m $d = 210$ mm
- $\Phi10 \ @100$ mm $m_{rd} = 69$ kNm/m $d = 210$ mm
- $\Phi10 \ @200 \ / \ \Phi16 \ @200$ mm $m_{rd} = 115$ kNm/m $d = 204$ mm

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The design shear force $V_d$ is equal to the column reaction force $N_d$ minus the applied load within the control perimeter $(g_d + q_d)A_c$.

In case of inner columns, the centroid of the column corresponds to the centroid of the control perimeter. Therefore, $\Delta e = 0$.

The distances $r_{sx}$ and $r_{sy}$ are the same as for the Level I approximation.

$k_{dg}$ is calculated at Level I.

### 3.5 Shear design inner column C5

#### Design shear force

$$A_i = b_i^2 + 2 \cdot b_i \cdot d_i + \frac{d_i^2}{4} \pi = 0.26^2 + 2 \cdot 0.26 \cdot 0.204 + \frac{0.204^2}{4} \pi = 0.21 \text{ m}^2$$

$$V_d = N_d - (g_d + q_d) \cdot A_i = 664 - 15.6 \cdot 0.21 = 661 \text{ kN}$$

#### Control perimeter

$$e_i = \frac{M_{d,\Delta e}}{V_d} = \frac{8 \cdot 10^6}{661 \cdot 10^3} = 12 \text{ mm} \quad b_z = \frac{4}{\pi} A_i = \frac{4}{\pi} 206 \cdot 10^3 = 513 \text{ mm}$$

$$k_e = \frac{1}{1 + e_i/b_z} = \frac{1}{1 + 12/513} = 0.98$$

$$b_y = k_e \cdot b_i \cdot (4 \cdot b_z + d_i \cdot \pi) = 0.98 \cdot (4 \cdot 206 + 204 \cdot \pi) = 1642 \text{ mm}$$

#### Rotations

$$r_{sx} = 1.32 \text{ m} \quad r_{sy} = 1.23 \text{ m}$$

$$b_z = 1.5 \cdot \sqrt{r_{sx} \cdot r_{sy}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \text{ m}$$

$$m_{sd,\Delta e} = \frac{V_d}{8} + \frac{M_{d,\Delta e} \cdot \Delta e}{2 \cdot b_z} = \frac{661}{8} + \frac{8}{2 \cdot 1.91} = 85 \text{ kNm/m}$$

$$m_{ed,\Delta e} = \frac{V_d}{8} + \frac{M_{d,\Delta e} \cdot \Delta e}{2 \cdot b_z} = \frac{661}{8} + \frac{1}{2 \cdot 1.91} = 83 \text{ kNm/m}$$

$$\psi_x = 1.5 \cdot \frac{r_{sx} \cdot f_{ud}}{d \cdot E_s} \left( \frac{m_{m,x}}{m_{sd,\Delta e}} \right)^{15} = 1.5 \cdot \frac{1.32 \cdot 435}{0.204 \cdot 200000} \left( \frac{85}{115} \right)^{15} = 0.0133 \Rightarrow \text{governing}$$

$$\psi_y = 1.5 \cdot \frac{r_{sy} \cdot f_{ud}}{d \cdot E_s} \left( \frac{m_{m,y}}{m_{sd,\Delta e}} \right)^{15} = 1.5 \cdot \frac{1.23 \cdot 435}{0.204 \cdot 200000} \left( \frac{83}{115} \right)^{15} = 0.0121$$

$$k_e = \frac{1}{1.5 + 0.6 \cdot \psi_x \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.0133 \cdot 204 \cdot 1.0} = 0.32 < 0.6$$
The punching shear strength of the concrete is not sufficient. Consequently, shear reinforcement is necessary.

Firstly, one has to check if the design shear force $V_d$ is smaller than the maximum punching strength $V_{rd,max}$. This is done assuming $k_{sys}=2$.

The design shear force $V_d$ is below the maximum punching strength $V_{rd,max}$. Therefore, the slab can be reinforced with shear reinforcement complying with detailing rules defined in subclause §7.13.5.3.

The bond strength is taken as $f_{bd} = 3 \text{ MPa}$ (according to MC 2010 for corrugated bars).

To avoid a failure outside the shear reinforced area, the outer perimeter need to have a minimal length. The design shear force can be reduced to account for the loads applied inside the outer perimeter. This effect is neglected as a safe estimate.

In this example, the calculating value of the effective depth $d_e$ is equal to the effective depth $d$ minus the concrete cover $c$ on the bottom surface of the slab.

$$d_{e,\text{calc}} = d - c = 204 - 30 = 174 \text{ mm}$$

Assuming a circular control perimeter for the estimation of the eccentricity, factor $k_e$ can be estimated as detailed in the right hand side column.

Possible shear reinforcement layout:

**Punching strength without shear reinforcement**

$$V_{rd,c} = k_y \frac{f_{ck}}{\gamma_c} b_y d_y = 0.32 \frac{\sqrt{30}}{1.5} 1642 \cdot 204 \cdot 10^{-3} = 390 \text{ kN} < V_d = 661 \text{ kN}$$

**Punching strength with shear reinforcement**

$$V_{rd,max} = k_y \cdot V_{rd,c} = 2 \cdot 390 = 781 \text{ kN} \geq \frac{f_{ck}}{\gamma_c} b_y d_y = 1223 \text{ kN}$$

$$V_{rd,max} = 781 \text{ kN} \geq V_d = 661 \text{ kN}$$

$$\sigma_{\text{red}} = \frac{E_y}{6} \left(1 + \frac{f_{bd}}{f_{yd}} \frac{d}{\phi_c} \right) = \frac{20000 \cdot 0.0133}{6} \left(1 + \frac{3}{435} \frac{204}{8} \right)$$

$$\sigma_{\text{red}} = 521 \text{ MPa} > f_{bd} = 435 \text{ MPa}$$

$$A_{sv} = \frac{V_d - V_{rd,c}}{k_y \sigma_{\text{red}} \sin \alpha} = \frac{(661 - 390) \cdot 10^3}{0.98 \cdot 435 \cdot \sin(90^\circ)} = 636 \text{ mm}^2$$

$$A_{sv,\text{max}} = \frac{0.5 \cdot V_d}{k_y \sigma_{\text{red}} \sin \alpha} = \frac{0.5 \cdot 661}{0.98 \cdot 435 \cdot \sin(90^\circ)} = 774 \text{ mm}^2 \Rightarrow \text{governing}$$

$$b_y = \frac{V_d}{k_y \frac{f_{ck}}{\gamma_c} d_{e,\text{calc}}} = \frac{661 \cdot 10^3}{0.32 \frac{\sqrt{30}}{1.5} 174} = 3258 \text{ mm}$$

$$b_y = \frac{V_d}{k_y \frac{f_{ck}}{\gamma_c} d_{e,\text{calc}}} = \frac{661 \cdot 10^3}{0.32 \frac{\sqrt{30}}{1.5} 174} = 3258 \text{ mm}$$

$$k_e = \frac{1}{1 + c_s/2r_{o}} = \frac{1}{1 + 12/(2 \cdot 519)} = 0.99$$

$$b_{o} = \frac{b_y}{k_e} = \frac{3258}{0.99} = 3296 \text{ mm}$$

$$\rho_{se} = \emptyset 8 \@ 100 \@ 100 = 0.50\%$$

$$A_{se} = \rho_{se} \left[ 4 \cdot b_y \cdot d_y + d_y^2 \cdot \pi - 4 \cdot b_y \cdot 0.35 d_y - (0.35 d_y)^2 \cdot \pi \right]$$

$$A_{se} = 0.005 \left[ 4 \cdot 260 \cdot 204 + 204^2 \cdot \pi - 4 \cdot 260 \cdot 0.35 \cdot 204 - (0.35 \cdot 204)^2 \cdot \pi \right]$$

$$A_{se} = 1263 \text{ mm}^2 > 774 \text{ mm}^2$$

$$b_{o} = 4 \cdot 700 + 174 \cdot \pi = 3347 \text{ mm} > 3296 \text{ mm}$$
The design shear force \( V_d \) is equal to the column reaction force \( N_d \) minus the applied load within the control perimeter \( (g_d + q_d)A_c \).

\[
\Delta \epsilon_x = \Delta \epsilon_y = \frac{3}{4} \left( \frac{b + d}{2} - b \right) = \frac{1}{4} \left( b + \frac{3}{2}d \right)
\]

The distances \( r_{sx} \) and \( r_{sy} \) are the same as for the Level I approximation.

In case of corner columns, the width of the support strip may be limited by the distance \( b_{sr} \).

3.6 Shear design corner column C1 and C3

**Design shear force**

\[
A_i = b^2 + 2 \cdot b \cdot \frac{d}{2} + \frac{d^2}{16} = 0.26^2 + 0.26 \cdot 0.210 + \frac{0.210^2}{16} = 0.13 \text{ m}^2
\]

\[
V_d = N_d - (g_d + q_d) \cdot A_i = 112 - 15.6 \cdot 0.13 = 110 \text{ kN}
\]

**Control perimeter**

\[
\Delta \epsilon_x = \Delta \epsilon_y = \frac{1}{4} \left( b + \frac{3}{2}d \right) = \frac{1}{4} \left( 260 + \frac{3}{2} \cdot 210 \right) = 144 \text{ mm}
\]

\[
\Delta e = \Delta \epsilon_i \cdot \sqrt{2} = 144 \cdot \sqrt{2} = 203 \text{ mm}
\]

\[
e_i = \left| \frac{M_x}{V_d - \Delta e} \right| = \left| \frac{33 \cdot 10^6}{110 \cdot 10^3 - 203} \right| = 97 \text{ mm} \quad b_i = \frac{4}{\sqrt{\pi}} A_i = \frac{4}{\sqrt{\pi}} \cdot 131 \cdot 10^3 = 408 \text{ mm}
\]

\[
k_i = \frac{1}{1 + e_i / b_i} = \frac{1}{1 + 97/408} = 0.81
\]

\[
b_x = k_i \cdot b = k_i \left( 2 \cdot b + \frac{d}{4} \right) = 0.81 \left( 2 \cdot 260 + 210 \cdot \frac{\pi}{4} \right) = 554 \text{ mm}
\]

**Rotations**

\( r_{sx} = 1.32 \text{ m} \quad r_{sy} = 1.23 \text{ m} \)

\( b_s = 1.5 \cdot \sqrt{r_{sx} \cdot r_{sy}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \text{ m} \)

\( b_x = 2 \cdot b = 2 \cdot 0.26 = 0.52 \text{ m} \Rightarrow \text{governing} \)

\[
m_{d,x} = \frac{V_{d,x}}{8} + \left| \frac{M_{d,x} - V_{d,x} \cdot \Delta \epsilon_i}{b_i} \right| = \frac{110}{8} + \left| \frac{25 - 110 \cdot 0.144}{0.52} \right| = 31 \text{ kN} < \frac{V_{d,x}}{2} = 110 \text{ kN}
\]

\[
m_{d,y} = \frac{V_{d,y}}{8} + \left| \frac{M_{d,y} - V_{d,y} \cdot \Delta \epsilon_i}{b_i} \right| = \frac{110}{8} + \left| \frac{22 - 110 \cdot 0.144}{0.52} \right| = 25 \text{ kN} < \frac{V_{d,y}}{2} = 110 \text{ kN}
\]
$k_{dg}$ is calculated at Level I.

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary.

Since no shear reinforcement has been used and $m_{ad} < m_{ld}$, integrity reinforcement needs to be provided.

For the design of the integrity reinforcement, the accidental load case can be used. Thus, the design load can be reduced.

$$
(g_s + q_s)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1 \text{kN/m}^2
$$

$$
V_{d,acc} = \left(\frac{g_s + q_s}{g_s + q_s}\right) V_s = \frac{10.1}{15.6} \cdot 110 = 71 \text{kN}
$$

The material properties can be found in chapter 5 of model code 2010.

Ductility class B : $(f_c/f_y)_k = 1.08$ and $\varepsilon_{uk} = 0.05$

It is assumed that only straight bars will be used, thus $\alpha = 0^\circ$.

With respect to integrity reinforcement, two restrictions should be fulfilled:

- the integrity reinforcement should at least be composed of four bars
- the integrity bars $\theta_{int}$ has to be chosen such that

$$
\begin{align*}
\psi_x &= 1.5 \cdot \frac{r_{sa} f_{cd}}{d} \left(\frac{m_{ad,x}}{m_{ld,x}}\right)^{1.5} = 1.5 \cdot \frac{1.32}{0.21} \cdot \frac{435}{200000} \left(\frac{55}{69}\right)^{1.5} = 0.0146 \Rightarrow \text{governing} \\
\psi_y &= 1.5 \cdot \frac{r_{sa} f_{cd}}{d} \left(\frac{m_{ad,y}}{m_{ld,y}}\right)^{1.5} = 1.5 \cdot \frac{1.23}{0.21} \cdot \frac{435}{200000} \left(\frac{55}{69}\right)^{1.5} = 0.0136 \\
k_y &= \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.0146 \cdot 210 \cdot 1} = 0.30 \leq 0.6
\end{align*}
$$

**Punching strength without shear reinforcement**

$$
V_{ad} = k_y \sqrt{\frac{f_y d_y}{\gamma_s}} b_y = 0.30 \sqrt{\frac{30}{1.5}} \cdot 554 \cdot 210 \cdot 10^{-3} = 127 \text{kN} > 110 \text{kN}
$$

**Integrity reinforcement**

$$
A_s = \frac{V_{d,acc}}{f_{cd} \cdot (f_c/f_y)_k \cdot \sqrt{1 - \left(\cos \alpha \cdot \frac{1 + 1.25 \varepsilon_{uo} d_{res}}{1 + 1.25 \cdot 0.05^{1.5}}\right)^2}} = \frac{71 \cdot 10^3}{435 \cdot 1.08 \cdot \sqrt{1 - \left(\cos(0) \cdot \frac{1 + 1.25 \cdot 0.05^{1.5}}{1 + 1.25 \cdot 0.05^{1.5}}\right)^2}} = 916 \text{mm}^2
$$

$\Rightarrow$ 2x2 Ø20 $A_s = 1257 \text{mm}^2$ (2 in each direction)

$$
0.12d_{int} = 0.12 \left( h - 2 \cdot c - \phi_{top} - \phi_{rinc} \right) = 0.12 \left( 250 - 2 \cdot 30 - 10 - 20 \right) = 19 \text{mm} \approx 20 \text{mm}
$$
The design shear force $V_d$ is equal to the column reaction force $N_d$ minus the applied load within the control perimeter $(g_d + q_d)A_c$.

$$
\Delta e_x = \frac{(b_x + d_x)\left(b_x + \frac{d_x}{2}\right) + 2\left(b_x + \frac{d_x}{2}\right)^2 - 1}{b_x + \frac{d_x}{2}}
$$

$$
\Delta e_y = \frac{1}{4} \frac{2b_y^2 + 6b_yd_y + 3d_y^2}{3b_y + 2d_y}
$$

The distances $r_{x,x}$ and $r_{y,y}$ are the same as for the Level I approximation.

In case of edge columns, the width of the support strip may be limited by the distance $b_{s_x}$.

### 3.7 Shear design edge column C2

**Design shear force**

$$
A_c = b_c^2 + 3 \cdot b_c \cdot \frac{d_c}{2} + \frac{d_c^2}{8} = 0.26^2 + \frac{3}{2} \cdot 0.26 \cdot 0.21 + \frac{0.21^2}{8} = 0.17 \, \text{m}^2
$$

$$
V_d = N_d - (g_d + q_d) \cdot A_c = 266 - 15.6 \cdot 0.17 = 263 \, \text{kN}
$$

**Control perimeter**

$$
\Delta e_x = \frac{1}{4} \frac{2b_x^2 + 6b_xd_x + 3d_x^2}{3b_x + 2d_x} = \frac{1}{4} \frac{2 \cdot 260^2 + 6 \cdot 260 \cdot 210 + 3 \cdot 210^2}{3 \cdot 260 + 2 \cdot 210} = 124 \, \text{mm}
$$

$$
\Delta e_y = 0 \, \text{mm} \quad \Delta e = \Delta e_x = 124 \, \text{mm}
$$

$$
\varepsilon = \frac{M_d}{V_d} - \Delta e = \left| \frac{42 \cdot 10^6}{263 \cdot 10^3} - 124 \right| = 35 \, \text{mm} \quad b_y = \frac{4}{\pi} A_c = \frac{4}{\pi} 167 \cdot 10^3 = 461 \, \text{mm}
$$

$$
k_y = \frac{1}{1 + \varepsilon_y/b_y} = \frac{1}{1 + 35/461} = 0.93
$$

$$
b_y = k_y \cdot b_y = 0.93 \left(3 \cdot b_y + \frac{d_y \cdot \pi}{2}\right) = 0.93 \left(3 \cdot 260 + 210 \cdot \frac{\pi}{2}\right) = 1031 \, \text{mm}
$$

**Rotations**

$$
r_{x,x} = 1.32 \, \text{m} \quad r_{x,y} = 1.23 \, \text{m}
$$

$$
b_x = 1.5 \cdot \sqrt{r_{x,x} \cdot r_{x,y}} = 1.5 \cdot \sqrt{1.32 \cdot 1.23} = 1.91 \, \text{m}
$$

$$
b_{w,x} = 3 \cdot b_x = 3 \cdot 0.26 = 0.78 \, \text{m} \Rightarrow \text{governing}
$$

$$
b_{w,y} = \frac{b_x + b_y}{2} = \frac{0.260 + 0.191}{2} = 0.199 \, \text{m} \Rightarrow \text{governing}
$$

$$
m_{x,x} = \frac{V_d}{8} + M_{x,x} \cdot \Delta e_x \cdot b_x = \frac{263}{8} + \frac{42 \cdot 263 - 0.124}{0.78} = 45 \, \text{kN}
$$

$$
m_{x,y} = \frac{V_d}{8} + M_{x,y} \cdot \Delta e_x \cdot 2b_x = \frac{263}{8} + \frac{0}{2 \cdot 1.09} = 33 \, \text{kN} < \frac{V_d}{4} = \frac{263}{4} = 66 \, \text{kN}
$$
$k_{dg}$ is calculated at Level I.

The punching shear strength of the concrete is not sufficient. Since the strength seems to be rather close to the design load, a level III approximation will be performed.

$$\psi_x = 1.5 \cdot \frac{r_{sh}}{d} \cdot \frac{f_{cd}}{E_s} \left( \frac{m_{ul,r}}{m_{ds,x}} \right)^{1/3} = 1.5 \cdot \frac{1.32}{0.21 \cdot 200000} \left( \frac{45}{69} \right)^{1/3} = 0.011$$

$$\psi_y = 1.5 \cdot \frac{r_{sh}}{d} \cdot \frac{f_{cd}}{E_s} \left( \frac{m_{ul,y}}{m_{ds,y}} \right)^{1/3} = 1.5 \cdot \frac{1.23}{0.21 \cdot 200000} \left( \frac{66}{69} \right)^{1/3} = 0.018 \Rightarrow \text{governing}$$

$$k_v = \frac{1}{1.5 + 0.6 \cdot \psi \cdot d \cdot k_{dg}} = \frac{1}{1.5 + 0.6 \cdot 0.018 \cdot 210 \cdot 1.0} = 0.27 < 0.6$$

**Punching strength without shear reinforcement**

$$V_{cd} = k_v \frac{f_{cd}}{\gamma} b_x d_x = 0.27 \sqrt{\frac{30}{1.5}} \cdot 1031 \cdot 210 \cdot 10^{-3} = 211 \text{kN} < 263 \text{kN}$$
The Level III calculations are based on the results of the linear-elastic finite element analysis.

From the results of the flexural analysis, one can obtain the distance between the center of the column and the point, at which the bending moments are zero.

The average moment in the support strip can be obtained by the integration of the moments at the strip section.

Since the flexural moments $m_{d,x}$ and $m_{d,y}$ at the support regions are negative, the absolute value of the twisting moment $m_{d,x}$ need to be subtracted so that the absolute value of $m_{sd,x}$ and $m_{sd,y}$ will be maximized.

$$m_{sd,x} = m_{d,x} - |m_{sd,y}|$$

$$m_{sd,y} = m_{d,y} - |m_{sd,x}|$$

The punching shear strength of the concrete is sufficient. Thus, no shear reinforcement will be necessary.

Since no shear reinforcement has been used and $m_{sd} < m_{rd}$, integrity reinforcement needs to be provided to prevent a progressive collapse of the structure.

For the design of the integrity reinforcement, the accidental load case can be used. Thus the design load can be reduced.

$$V_{d,acc} = (g_d + q_d)_{acc} = 1.0(6.25 + 2) + 0.6 \cdot 3 = 10.1 \text{ kN/m}^2$$

$$V_{sd} = \frac{(g_d + q_d)_{acc}}{g_d + q_d} \cdot V_d = \frac{10.1}{15.6} \cdot 263 = 171 \text{ kN}$$

4 Level III of approximation (detailed design or assessment of existing structure)

$$r_{sd,x} = 0.64 \text{ m} \quad r_{sd,y} = 1.18 \text{ m}$$

$$b_x = 1.5 \cdot \sqrt{r_{sd,x} \cdot r_{sd,y}} = 1.5 \cdot \sqrt{0.64 \cdot 1.18} = 1.30 \text{ m}$$

$$b_{w,x} = 3 \cdot b_x = 3 \cdot 0.78 = 0.87 \text{ m}$$

$$b_{w,y} = \frac{b_x + b_y}{2} = \frac{0.26 + 1.30}{2} = 0.78 \text{ m}$$

$$m_{sd,x} = 24 \text{ kN/m} \quad m_{sd,y} = 43 \text{ kN/m} \quad \text{(average value on support strip)}$$

$$r_{sd,x} = 0.64 > \frac{2}{3} b_{w,x} = 0.52 \text{ m} \quad r_{sd,y} = 1.18 > \frac{2}{3} b_{w,y} = 0.52 \text{ m}$$

$$\psi_x = 1.2 \cdot \frac{r_{sd,x} \cdot f_{sd}}{d \cdot E_s} \left( \frac{m_{sd,x}}{m_{rd,x}} \right)^{1/3} = 1.2 \cdot \frac{0.64}{0.21} \cdot \frac{435}{0.21 \cdot 200000} \left( \frac{24}{69} \right)^{1/3} = 0.0016$$

$$\psi_y = 1.2 \cdot \frac{r_{sd,y} \cdot f_{sd}}{d \cdot E_s} \left( \frac{m_{sd,y}}{m_{rd,y}} \right)^{1/3} = 1.2 \cdot \frac{1.18}{0.21} \cdot \frac{435}{0.21 \cdot 200000} \left( \frac{43}{69} \right)^{1/3} = 0.0073 \Rightarrow \text{governing}$$

$$k_y = \frac{1}{1 + 0.6 \cdot \psi \cdot d \cdot k_{dI}} = \frac{1}{15 + 0.6 \cdot 0.0073 \cdot 210 \cdot 1.0} = 0.41 < 0.6$$

$$V_{rd} = k_y \sqrt{\frac{f_{sd}}{\gamma}} b_d = 0.41 \sqrt{\frac{30}{15}} \cdot 1047 \cdot 210 \cdot 10^{-3} = 332 \text{ kN} > 263 \text{ kN}$$

Integrity reinforcement

$$A_s = \frac{V_{d,acc}}{f_{sd} (f / f_0)^2 \cdot \sqrt{1 - \left( \frac{\cos \alpha}{1 + 1.25 c_s^3} \right)^3}} = \frac{171 \cdot 10^3}{435 \cdot 1.08 \cdot \sqrt{1 - \left( \frac{\cos (0)}{1 + 1.25 \cdot 0.05^3} \right)^3}} = 2194 \text{ mm}^2$$

$$\Rightarrow 2 \times 3 \times 20 + 3 \times 16 A_s = 2488 \text{ mm}^2 \quad (3 \text{ in each direction})$$

$$0.12 d_{sd} = 0.12 (h - 2 \cdot c - \phi_{op} - \phi_{con}) = 0.12 (250 - 2 \cdot 30 - 10 - 20) = 19 \text{ mm} \approx 20 \text{ mm}$$
The material properties can be found in chapter 5 of model code 2010.

Ductility class B : \((f_y/f_y)_k = 1.08\) and \(\varepsilon_u k = 0.05\)

It is assumed that only straight bars will be used, thus \(\alpha = 0^\circ\).

With respect to integrity reinforcement, two restrictions should be fulfilled:
- the integrity reinforcement should at least be composed of four bars
- the diameter of the integrity bars \(\sigma_{int}\) has to be chosen such that \(\sigma_{int} \leq 0.12 \cdot d_{res}\)

Corners of walls should be checked following the same methodology.

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