

## CONCRETE TIE MODEL FOR THE FLEXURAL BEHAVIOR OF RC AND PC SECTIONS

*Stéphane Rossier<sup>1</sup> and Olivier Burdet<sup>2</sup>*

*Swiss Federal Institute of Technology, Department of Civil Engineering,  
Institute of Reinforced and Prestressed Concrete  
CH-1015 Lausanne*

### SUMMARY

The method described in this paper is capable of determining the flexural behavior of RC and PC sections. It uses a "slice" discretization of the cross section and thus, it is applicable to any concrete section, without shape restriction. The described tie model used for the calculation of each slice is destined to be finally implemented in a shell FE package.

**Keywords:** serviceability behavior, numerical modeling, nonlinear response, cracking, creep, finite element calculation, RC & PC tie

### 1. INTRODUCTION

Over the past thirty years, several post-tensioned bridges have exhibited an unsatisfactory long-term behavior under service loads, characterized by a non-stabilization of deformation and increased cracking over time. While explanations are available [1], some effects cannot be easily taken into account using beam analysis only. It is expected that the finer degree of modeling offered by shell analysis will lead to a better understanding of the actual behavior.

The goal of the research project is to develop a computation tool suitable for use by civil engineers to evaluate the effect of cracking and long-term effects (creep & shrinkage). This will be achieved by implementing a new material model into an existing commercial shell FE program.

The CEB-FIP moment-curvature relationship is a well established tool for the computation of long-term deflections of concrete structures. FE programs using this relationship have already been developed and successfully used for the computation of beam-type structures [2]. Its validity is however limited to beam-type structures. This research proposes an enhanced concrete tie model, derived from another CEB contribution [3]. The enhanced concrete tie model accurately describes the behavior of a concrete tie in tension or compression, as well as the behavior of a "slice" of concrete in a section subjected to bending with and without axial force. This allows its use to

---

<sup>1</sup> civil engineer EPFL, PhD student

<sup>2</sup> PhD, civil engineer EPFL-SIA

describe the in-plane properties of shell elements, as they appear, for example, in the web or flange of a box-girder section. This model has been extensively compared to the results given by the CEB MC relationship and has given excellent results. These results, as well as the method, are described in this paper.

## 2. PHYSICAL MODELS

### 2.1. CEB-FIP Moment-curvature relationship

This relationship gives the mean curvature of an RC beam subjected to bending moment with or without an axial compression force. It gives accurate results for compact sections with a vertical symmetry axis. On the other hand, it does not include shear effects. Furthermore axial tension forces are not handled.

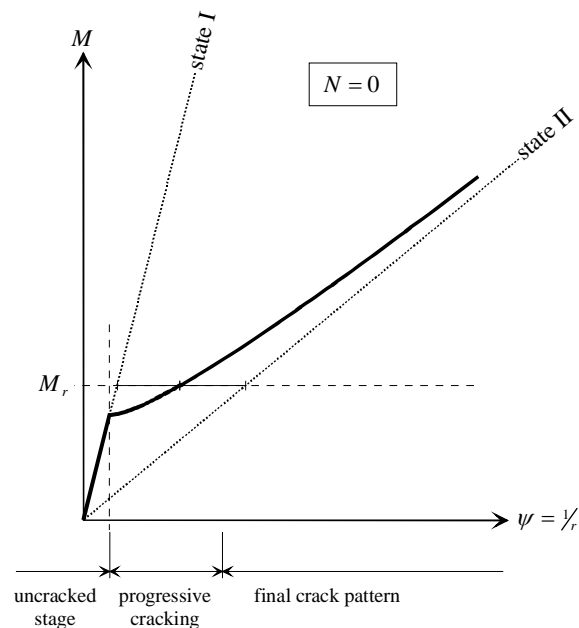


Figure 1. CEB-FIP Moment-curvature relationship

$$\psi_m = \psi_2 - \beta \cdot (\psi_{2r} - \psi_{1r}) \cdot \frac{M_r}{M} \geq \psi_1$$

$M, N$  = bending moment and normal force applied to the section

$\psi_m$  = mean curvature under  $M, N$

$M_r$  = cracking moment of the section under  $N$

$\psi_1, \psi_2$  = state I,II curvatures under  $M, N$

$\psi_{1r}, \psi_{2r}$  = state I,II curvatures under  $M_r, N$

$\beta$  = uncracked concrete contribution coefficient  
= 0.8 for first loading

= 0.5 for long term loading

## 2.2. Concrete tie model

The enhanced concrete tie model is derived from the relationship proposed by Sippel in [3]. Two modifications are introduced.

In the original model, the stabilized-crack branch of the relationship is parallel to the state II. Since the moment-curvature relationship is asymptotical to the state II, the stabilized-crack branch has been modified as show in figure 2.

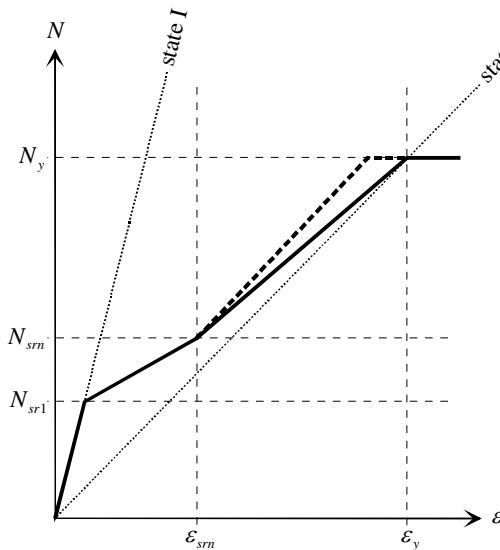


Figure 2. Modification of the final crack pattern stage

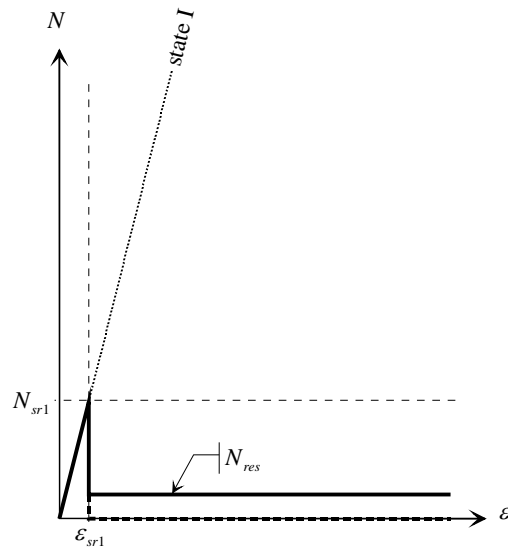


Figure 3. Residual tensile strength of cracked non reinforced slice

In low reinforced concrete slices, the tensile strength after the first crack occurs is equal to the yielding force of the reinforcement. In very low or non reinforced concrete slices, a residual tensile strength is defined as a percentage of the tensile strength of the slice (fig. 3). This applies especially to the intermediate zones of the section where there is no particular reinforcement and the strains are imposed by the main reinforcement at the top or at the bottom of the section.

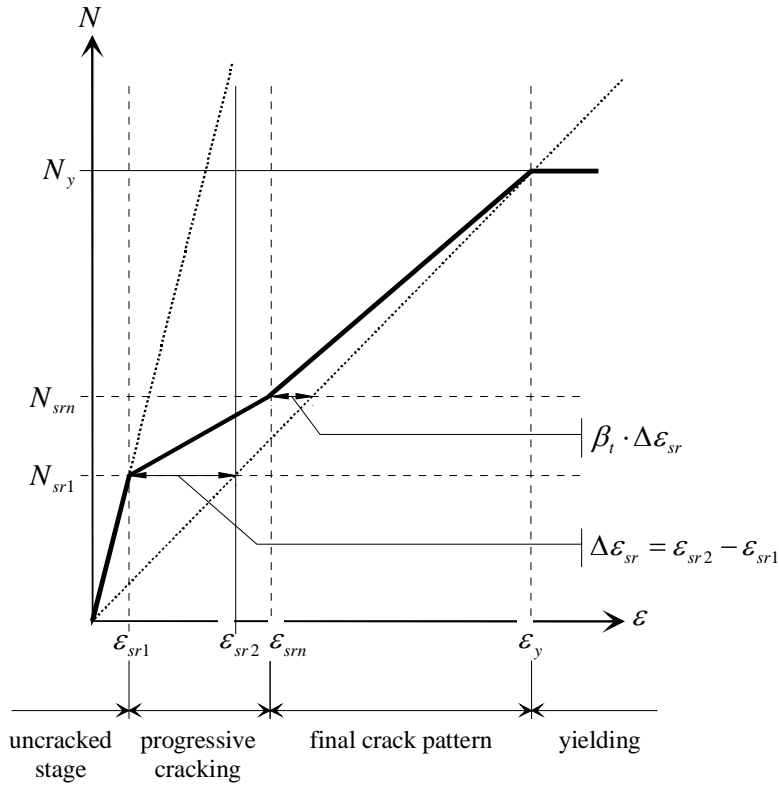


Figure 4. Modified relationship for a typical reinforced concrete section

Hence the following formulation, for a reinforced concrete slice at service state:

$f_{ct,fl}$  = mean flexural tensile strength of concrete

$$f_{ct,fl5\%} = 0.75 \cdot f_{ct,fl}$$

$$f_{ct,fl95\%} = 1.25 \cdot f_{ct,fl}$$

$A_c$  = raw concrete section area

$A_s$  = reinforcement section area

$E_s$  = E-modulus of steel

$E_c$  =  $E_c(t)$  E-modulus of concrete

$EA_I$  =  $E_c \cdot (A_c - A_s) + E_s \cdot A_s$  (stiffness of the uncracked slice - state I)

$EA_{II}$  =  $E_s \cdot A_s$  (stiffness of the cracked slice - state II)

$N$  = force in the RC slice

$\varepsilon$  = strain in the RC slice

**uncracked Stage**, including compression:

$$N = EA_I \cdot \varepsilon \quad \text{if} \quad \varepsilon < \varepsilon_{sr1}$$

with  $\varepsilon_{sr1} = k \cdot f_{ct,fl 5\%} / E_c$

$k$  = reduction factor for the tensile strength under long term loading

**progressive cracking:**

$$N = N_{sr1} + \frac{N_{srn} - N_{sr1}}{\varepsilon_{srn} - \varepsilon_{sr1}} \cdot (\varepsilon - \varepsilon_{sr1}) \quad \text{if} \quad \varepsilon_{sr1} < \varepsilon < \varepsilon_{srn}$$

with  $N_{sr1} = \varepsilon_{sr1} \cdot EA_I$  (first crack)

$N_{srn} = N_{sr1} \cdot f_{ct,fl 95\%} / f_{ct,fl 5\%}$  (last crack)

$\varepsilon_{sr2} = N_{sr1} / EA_{II}$

$\varepsilon_{srn} = N_{srn} / EA_{II} - \beta_t \cdot (\varepsilon_{sr2} - \varepsilon_{sr1})$

$\beta_t = 0.40$  first loading

$\beta_t = 0.25$  long term loading or cyclic loading

**final crack pattern:**

$$N = N_{srn} + \frac{N_y - N_{srn}}{\varepsilon_y - \varepsilon_{srn}} \cdot (\varepsilon - \varepsilon_{srn}) \quad \text{if} \quad \varepsilon_{srn} < \varepsilon < \varepsilon_y$$

with  $\varepsilon_y = f_y / E_s$

$N_y = f_y \cdot A_s$

**Reinforcement yielding:**

$N = N_y$  if  $\varepsilon_y < \varepsilon$

**Two more conditions must be checked.** First:

If  $\varepsilon > \varepsilon_{sr1}$  then  $N \leq N_y$

And second, the residual tensile force of the slice:

If  $\varepsilon > \varepsilon_{sr1}$  then  $N \geq N_{res} = 12\% \cdot N_{sr1}$

The value of 12% has been calibrated by testing the numerical model.

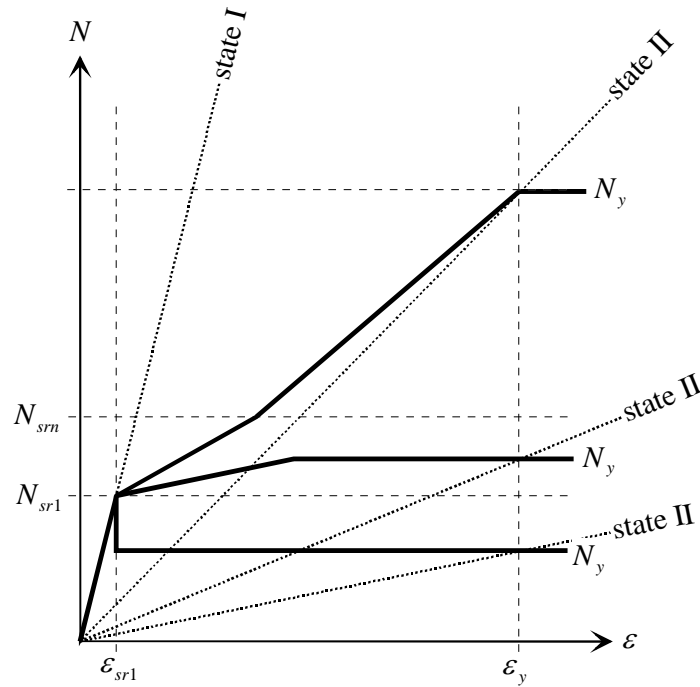


Figure 5. Force-strain relationship for different reinforcement levels of the same concrete section

### 2.3. Implementation

The RC or PC section is discretized into slices parallel to the neutral axis. The reinforcement is distributed in the concerned slices. The tie model is applied to each slice.

Since the force-strain relationship is a global model considering the reinforced concrete as an homogenous material, the discretization has to be rather rough. To avoid scale effects and to take in account the reinforcement bars spacing, we are going to consider an equivalent slice with a reduced concrete area.

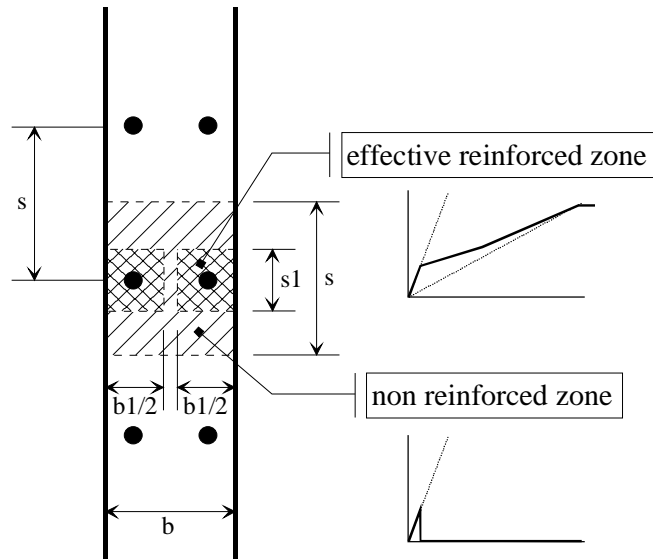


Figure 5. Effective application zone of the concrete tie relationship

### 3. COMPARISON MODEL

#### 3.1. CEB-FIP moment curvature model

...

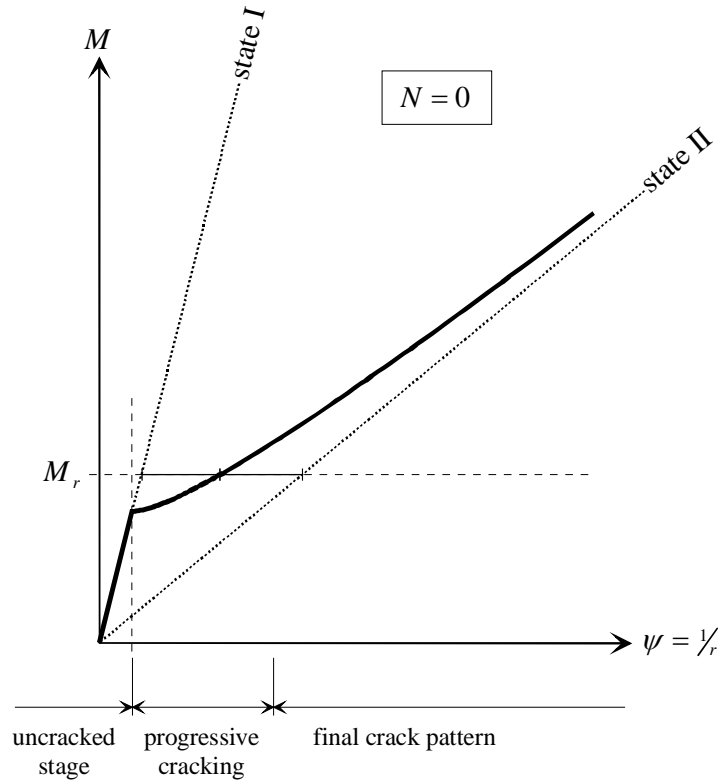


Figure x. CEB-FIP Moment-curvature relationship

$$\psi_m = \psi_2 - \beta \cdot (\psi_{2r} - \psi_{1r}) \cdot \frac{M_r}{M} \geq \psi_1$$

$M, N$  = bending moment and normal force applied to the section

$\psi_m$  = mean curvature under  $M, N$

$M_r$  = cracking moment of the section under  $N$

$\psi_1$  = state I curvature under  $M, N$

$\psi_2$  = state II curvature under  $M, N$

$\psi_{1r}$  = state I curvature under  $M_r, N$

$\psi_{2r}$  = state II curvature under  $M_r, N$

$\beta$  = uncracked concrete contribution coefficient

= 0.8 for first loading

= 0.5 for long term loading

### 3.2. Comparison criterion for large scale parametric studies

...



## 4. RESULTS

...

### 4.1. Comparison with CEB-FIP moment-curvature model

...

#### 4.1.1. Reconstructed moment-curvature relationships

...

#### 4.1.2. Application of global comparison criterion

...

#### 4.1.3. Parametric study on 3000 sections

...

## 5. CONCLUSIONS

The concrete tie model has been implemented in the finite element program. In the coming months, the program will be applied to special cases of long-term behavior. In particular, the effect of shrinkage-induced cracking of deck slabs will be investigated.

## 6. ACKNOWLEDGEMENTS

...

## 7. REFERENCES

- [1] R. Favre, O. Burdet, H. Charif, M. Hassan, I. Markey : *Enseignements tirés d'essais de charge et d'observations à long terme pour l'évaluation des ponts et le choix de la précontrainte*, Rapport OFR N° 514, VSS, Zürich, 1995.
- [2] A. Bouberguig, S. Rossier, R. Favre, H. Charif : *Calcul non linéaire du béton armé et précontraint*, Revue Française du Génie Civil, 503-568 Vol 1 No 3, Hermes, Paris, 1997
- [3] CEB Information Bulletin n° 235, *Serviceability Models*, Lausanne, April 1997.
- [4] S. Rossier : *AlphaFlex (software for nonlinear calculation of RC & PC sections)*, IBAP – EPFL, Lausanne, 1998

As the goal of this first step is to demonstrate the concordance between the proposed model and the CEB-FIP moment-curvature relationship, we are going to calculate only sections with vertical symmetry axis. This is not a limitation of the method.

The base of the model lies in the local application of a concrete tie relationship. The cross section is discretized into slices parallel to the neutral axis. The reinforcement is distributed in each concerned slice. The strains are considered constant in each slice and linear accross the section.