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Influence of Tensile Properties of UHPFRC on Size Effect in Bending

Summary

Bending behaviour of members without ordinary reinforcement made of quasi-brittle materials (such as concrete or fibre-reinforced concrete) is subjected to size effect both in terms of strength and ductility. For a typical ultra-high performance fibre-reinforced concrete (UHPFRC), size effect on bending strength is less significant. In this paper, taking advantage of an analytical approach, this behaviour is confirmed if large tensile strains are developed in UHPFRC before beginning of tension softening. However, it is also shown that changes in the size of the member have a significant influence on the ductility in bending. Both results are in agreement with available experimental data. A parametric study on size effect in bending is finally performed by varying the tensile strain before beginning of tension softening and the initial slope of the tension softening law. On that basis, some considerations on the modelling and design of such members are given.

Keywords: *UHPFRC, bending, size effect, tension strain hardening, tension softening*

1 Introduction

Ultra-high performance fibre-reinforced concrete (UHPFRC) is a new type of fibre-reinforced material, having a high quantity of fibre reinforcement (usually more than 2 % in volume of metallic fibres) and a very dense matrix. UHPFRC has high compressive and tensile strengths and a ductile behaviour in tension. An interesting possibility to take advantage of UHPFRC mechanical properties in tension is to use it for thin members in bending without ordinary reinforcement. This is justified because of the significant ductility of such members and by their limited size effect on strength in bending.

Contrary to the rather ductile behaviour of UHPFRC in tension, the tensile behaviour of ordinary concrete (OC) and of fibre-reinforced concrete (FRC) is characterized by a linear elastic phase followed, after cracking, by a tension softening phase with localisation of the strains in a single crack (Figure 1a and 1b). This difference in the tensile behaviour is due to the fact that after cracking of the UHPFRC matrix, fibres can carry larger tensile forces than the matrix itself. As a consequence, a large number of cracks develop in UHPFRC and cracking of the matrix is not directly followed by strain localisation. This phenomenon, often named multi-microcracking, results in a strain hardening behaviour characterized by a limited or no stress increase with development of large tensile strains [1] (Figure 1a). This phase will be named in this paper as *pseudo-plastic* behaviour.

The softening behaviour, characterizing the response after crack localisation, can be described in terms of the stress-crack opening relationship (Figure 1b). According to the assumptions of the Fictitious Crack Model [2], the area below the stress-crack opening relationship is the fracture energy G_F. UHPFRC is characterized by a fracture energy which is up to several hundred times that of an OC. Another significant difference between these materials is the slope of the stress-crack opening law, much larger for OC or FRC than for UHPFRC (Figure 1b).

a) b) c)

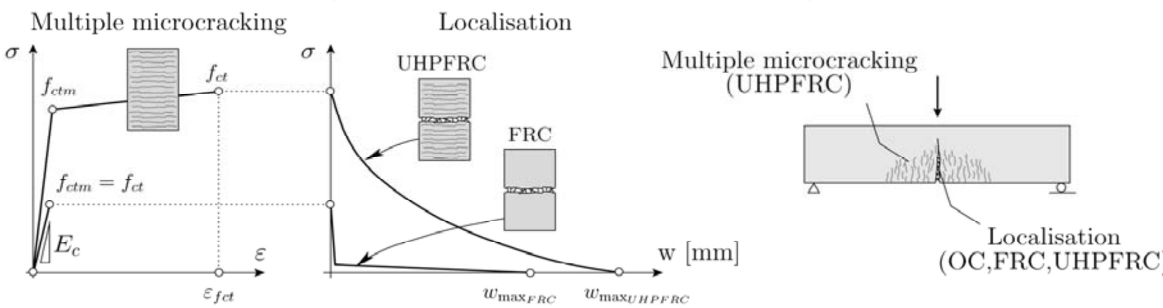

Figure 1: Qualitative comparison between the tensile behaviour of FRC and UHPFRC: a) stress-strain relationship before crack localisation; b) stress-crack opening relationship (tensile softening); c) beam in bending with crack localisation and a microcracked region.

From the point of view of fracture mechanics, both OC and FRC can be considered as quasi-brittle materials. Thus, their bending behaviour is sensitive to size effect both in terms of strength and ductility [3]. It will be shown in this paper that for quasi brittle materials that have a pseudo-plastic phase with sufficient tensile strain capacity (and for a range of thicknesses interesting for structural applications), the size effect on the bending strength is practically negligible and less significant than the scatter of test results.

This paper focuses on the behaviour of simply supported beams failing in bending. Failure is assumed to occur by the development of a discrete crack in a multi-microcracked zone (Figure 1c). The analyses presented in the paper are performed using an analytical model developed by the first author of this paper [4] based on equilibrium and energy-balance conditions. The accuracy of the analytical model has been checked against the results of a finite element model implementing the same hypotheses (fictitious crack [2] and pseudo-plastic behaviour in tension) and against experimental results on beams tested at the EPFL by the authors of this paper [4] and taken from the literature [5], Figure 2. The results shown in Figure 2a and 2b were obtained for beams in three-point bending with 420 mm span, whereas results in Figure 2c refer to beams in four-point bending with 300 and 900 mm span.

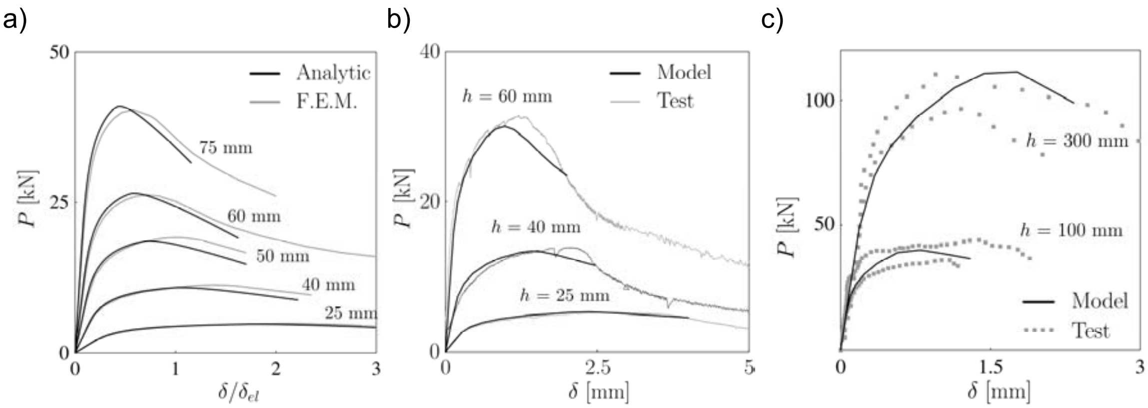

Figure 2: Comparison of theoretical and experimental behaviour for beams in bending: a) results of analytical and FEM models for UHPFRC beams; b) model and test results for UHPFRC beams taken from [4]; c) model and test results for beams made of engineered cementitious composite (ECC), [5].

To compare the bending response of beams made of different materials, the terms *equivalent bending stress* and *equivalent bending strength* are used in this paper. The equivalent bending stress is defined as:

$$\sigma_{equ} = \frac{M}{I_{z,el}} \cdot y \tag{1}$$

where M is the bending moment, $I_{z,el}$ is the moment of inertia of the elastic uncracked section and y is the distance between the centre of gravity and the outermost tensile fibre of the section. The equivalent bending strength, f_{equ}, is given by Eq. (1) with $M = M_{max}$, where M_{max} is the maximum bending moment carried by the section. Although σ_{equ} and f_{equ} have no physical meaning once tensile strength f_{ctm} is exceeded, they are used in this paper as reference values.

2 Size effect on strength and ductility of members in bending

As previously introduced, the tensile properties of a material have a significant influence on its bending behaviour. This phenomenon is investigated in this section with reference to bending strength and ductility considering the case of a beam subjected to three-point bending (Figure 1c). The slenderness of the beam is kept constant, with a span to depth ratio L / h equal to 8. The depth of the beams is varied from 25 to 500 mm to investigate size effect in a range of thicknesses interesting for structural applications. Three different materials are considered (Figure 3):

1. UHPFRC BSI-Céracem [6], which is characterized by a Young modulus of 60 GPa, a tensile strength of 9 MPa, a pseudo-plastic plateau extending up to a strain of 2.5 ‰ and an initial strain softening slope $F = d\sigma/dw$ equal to 6.8 MPa/mm (Figure 3a);
2. a hypothetic material with the same elastic and softening behaviour as the UHPFRC, but without the pseudo-plastic plateau;

3. an ordinary FRC with a Young modulus of 30 GPa, a tensile strength of 3 MPa and F equal to 20 MPa/mm (Figure 3b).

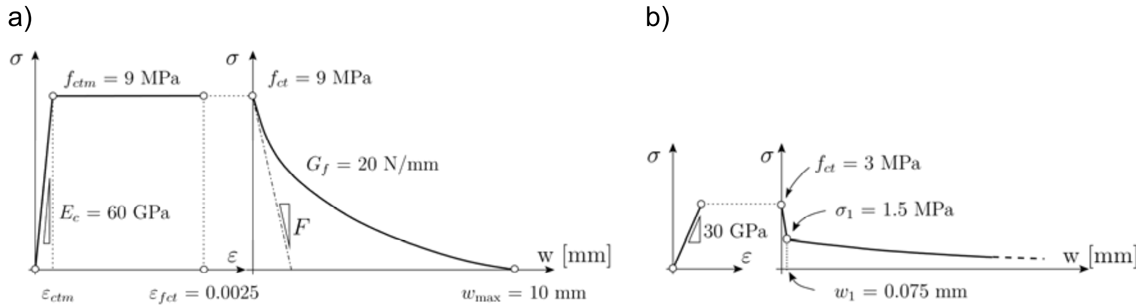

Figure 3: Tensile behaviour of: a) the UHPFRC tested at the EPFL; b) a FRC.

The results of the analyses are plotted in Figure 4 in terms of equivalent bending stress versus nominal deflection (defined as the ratio between the deflection δ at mid-span of the beam and its value δ_{el} at cracking of the matrix). Comparing the peaks of the different curves, it can be noted that for UHPFRC (Figure 4a) the equivalent bending strength depends only slightly on the size of the elements, whereas for FRC (Figure 4c) the size effect on the bending strength of the member is more significant. Figure 4 also shows that the ductility of the members strongly depends on their size and on the mechanical properties of the materials. In terms of ductility, the size effect is qualitatively similar for all materials investigated.

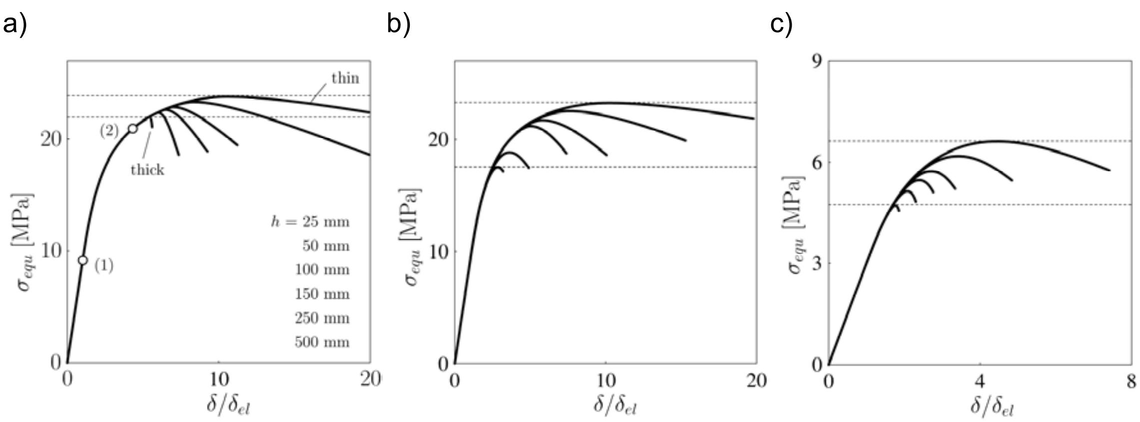

Figure 4: Bending behaviour of beams made of: a) UHPFRC with pseudo-plastic phase; b) UHPFRC without pseudo-plastic phase; c) FRC. In a), point 1 corresponds to matrix cracking and point 2 corresponds to the end of pseudo-plastic phase.

It is interesting to note that for the material with the same softening as UHPFRC but without the pseudo-plastic plateau, the behaviour is similar to that of UHPFRC for thin members (25 to 150 mm), with a limited size effect on strength. On the contrary, it is similar to that of ordinary FRC for thicker elements, for which size effect becomes more pronounced (Figure 4b, 5a).

In Figure 5a, the results of the simulations are plotted for the various materials in terms of the ratio between the equivalent bending strength and the tensile strength of the material versus the thickness of the elements. The upper value on the vertical axis ($f_{equ} / f_{ct} = 3$) corresponds to a perfectly-plastic behaviour in tension. Results are given also for an ordinary concrete that, for significant sizes, can be considered as the limiting case of an elastic-perfectly brittle material ($f_{equ} / f_{ct} \rightarrow 1$). This diagram shows that for UHPFRC size effect is clearly less significant than for FRC and OC.
Figure 5b presents the experimental results obtained at the EPFL [4] for UHPFRC beams of various thicknesses. The experimental results confirm that, for thicknesses ranging from 25 to 75 mm, the size effect on the strength cannot be clearly appreciated (the scatter of the results is larger than size effect). The results of the model are also confirmed by similar experimental results presented by other authors [5, 7] who investigated materials with pseudo-plastic tensile behaviour in beams with thicknesses up to 300 mm.

a) b)

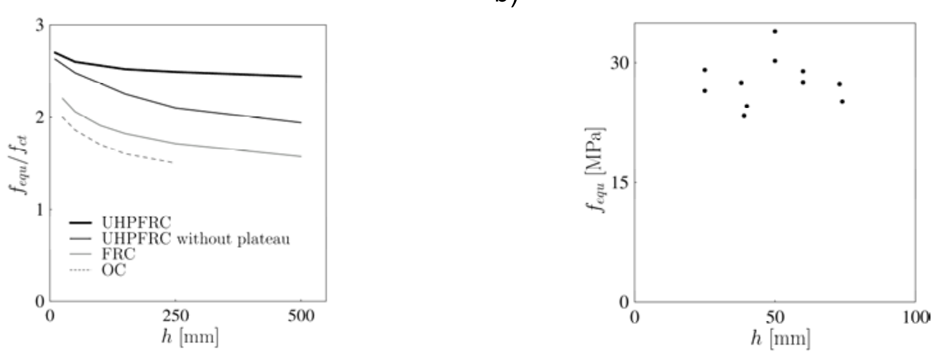

Figure 5: a) Theoretical size effect in UHPFRC, UHPFRC without pseudo-plastic plateau, FRC and OC; b) experimental size effect on bending strength for UHPFRC beam [4].

3 Influence of pseudo-plastic phase and tension softening on the bending behaviour

The limited influence of size effect on the bending strength of UHPFRC can be explained by the fact that most of the bending strength is activated during development of the pseudo-plastic phase in concrete (Figure 6a), whose behaviour is size-independent. Analytically, it can be shown that for a typical UHPFRC (ε_{fct} = 2 – 3 ‰) the equivalent bending stress attained before beginning of tension softening (Figure 6a) is between 2.2 - 2.5 times the tensile strength f_{ct}, [4]. The additional increase in strength, developed with a local tension softening (Figure 6b), is size-dependent. This increase is limited however (typically varying between 0.2 f_{ct} to 0.4 f_{ct} , corresponding to the variation of σ_{equ} between point 2 and the maximal strengths in Figure 4.a).
In addition to the strain developed prior to tension softening, the slope of the stress-crack opening law also plays a major role. For instance, considering a material with the same softening as a UHPFRC, it was shown in the previous section that size effect on the strength of thin elements is strongly reduced even without a pseudo-plastic phase. This can be

understood with the help of Figure 6c: if the slope of the softening branch is small, the stress distribution in the tensile zone is similar to that obtained with a pseudo-plastic behaviour in tension (Figure 6a). As a consequence, for thin elements a pseudo-plastic behaviour or a strain softening with a small slope lead to similar results with respect to size effect.

a) b) c)

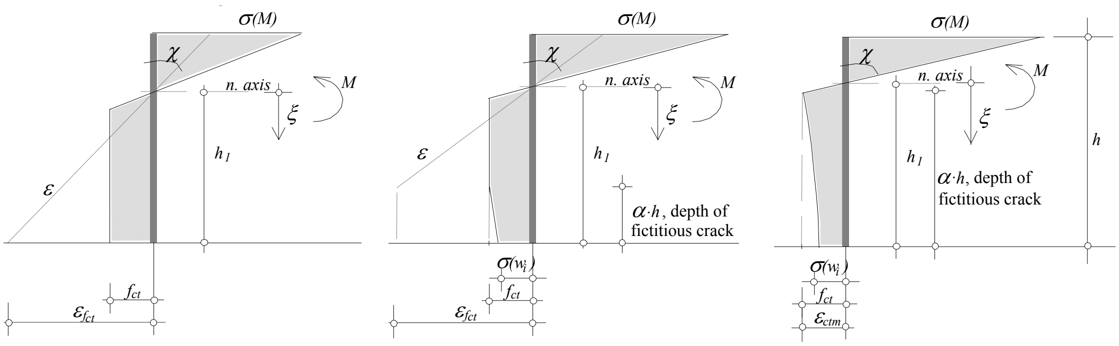

Figure 6: Stress distribution in the section developing a fictitious crack: a) development of quasi-plastic phase in tension; b) local softening due to opening of a fictitious crack (with a part of the height of the section in quasi-plastic phase); c) softening due to opening of a fictitious crack without quasi-plastic phase in tension.

For modelling and design purposes, however, the two phenomena differ. In case of a dominant pseudo-plastic behaviour, the sectional and the structural response can be modelled using a continuum approach based on stress-strain relationships [8]. On the contrary, when the tensile softening behaviour dominates, strain localisation develops and models based on fracture mechanics theories need to be used.

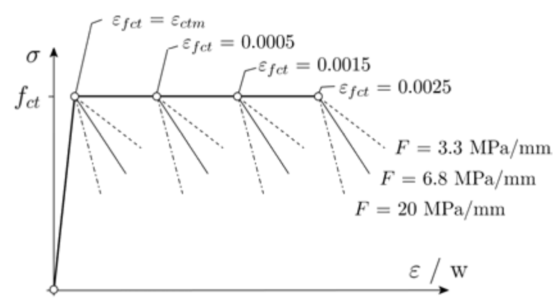

Figure 7: Variation in material properties in tension: strain ε_{fct} at the end of the pseudo-plastic phase and initial slope of the stress-crack opening law.

In order to understand the relative importance of the pseudo-plastic phase and of the softening behaviour of a material on the strength and ductility of a member, a parametric analysis has been performed. The strain ε_{fct} prior to tension softening and the initial slope of the stress-crack opening law are the main parameters for this study. The choice of the latter parameter is justified since, for most structural applications, the bending strength is reached with small crack openings and thus the initial slope of the stress crack opening relationship is more significant than the value of the fracture energy G_F [4]. The parametric study is performed considering a UHPFRC with the same material properties as those detailed in

Section 2. The values of ε_{fct} and of the initial softening slope are varied according to Figure 7. The values chosen for the softening slope F are: 3.3 MPa/mm, 6.8 MPa/mm, representing the UHPFRC used in this study, and 20 MPa/mm, which reasonably approximates the initial slope for an ordinary FRC.

The results of the analyses are presented in Figure 8. The nine curves in plot a) were obtained considering a constant thickness of 50 mm. This figure shows that for increasing values of the pseudo-plastic strain prior to tension softening (black curves) the contribution of the softening slope on the structural response becomes less significant.

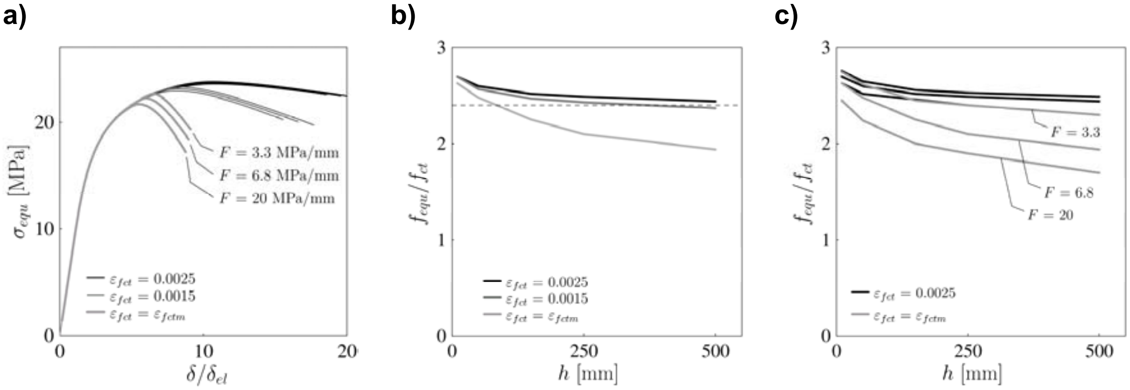

Figure 8: Influence of pseudo-plastic plateau and strain softening slope on the bending response: a) bending stress for a beam with h = 50 mm, varying ε_{fct} and F; b) influence of ε_{fct} on size effect, with constant F; c) influence of F on size effect, for ε_{fct} = 2.5 ‰ (solid lines) and $\varepsilon_{fct} = \varepsilon_{el}$ (grey lines).

On the contrary, for small values of ε_{fct} (grey lines) variations of the softening slope have a significant influence on the bending strength (Figure 8a). In Figure 8b and 8c, the analysis is extended to thicknesses varying from 25 to 500 mm. The influence of ε_{fct} for a constant tension softening slope is shown in Figure 8-b. The results clearly show that the size effect on the bending strength is very limited in the case of a pseudo-plastic plateau with strains up to 2.5‰. The same applies for ε_{fct} = 1.5‰, whereas for smaller values of ε_{fct} the size effect is more pronounced. However, if the softening slope is small, the bending strength of elements without softening plateau (Figure 8c, grey line with F = 3.3 MPa/mm) is similar to that of elements with a pseudo-plastic plateau (black lines).

4 Conclusions

This paper investigates size effect of UHPFRC members in bending. A parametric study has been performed by varying the tensile strain prior to development of a localised crack (pseudo-plastic phase) and the initial slope of the tension softening law. The results have been compared to those obtained on members made of FRC and OC. On that basis, the following conclusions are drawn:

1 in a range of thicknesses interesting for structural applications (25 to 500 mm), size effect on bending strength is limited for a typical UHPFRC, characterized by a pseudo-plastic behaviour in tension with a deformation capacity of 2 to 3 ‰;

2 for very thin members (25 to 75 mm) it is theoretically and experimentally demonstrated that size effect on bending strength is practically negligible;
3 in presence of a significant pseudo-plastic behaviour, the influence of the post-peak tension softening on bending strength is not very significant, because most of the bending strength is developed while concrete is in the pseudo-plastic tensile phase. The post-peak tensile behaviour is however relevant to the ductility in bending;
4 in the case of limited or no pseudo-plastic phase, the bending response of UHPFRC elements is similar to that of ordinary FRC. However, for thin members the small value initial slope of the tension softening stress-crack opening law leads to a rather ductile behaviour and to a limited size effect on the strength, even if there is no pseudo-plastic phase;
5 in case of a significant pseudo-plastic behaviour, modelling and design can be performed with simple approaches based on continuum mechanics and stress-strain relationships.

5 References

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